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KEY TO THE PROBLEMS IN AVERY'S SCHOOL PHYSICS

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IN MEMORIAM
J. Henry Senger

KEY TO THE PROBLEMS

IN

AVERY'S SCHOOL PHYSICS

UNIV. OF CALIFORNIA

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KEY

TO THE

PROBLEMS IN AVERY'S SCHOOL PHYSICS.

CHAPTER I.

Page 20.

8. See Exercise 11, page 95.

9. The number of inches in a meter is given on page 18; 39.37 inches. The number of millimeters in an inch is the reciprocal of the value of a millimeter as given on the same page; $1 \div 0.03937 = 25.39$.

10. Four times 250 cu. cm. equals 1 liter.

11. See § 21.

1.0567 qts. = 1,000 cu. cm.

1 qt. = 1,000 cu. cm. $\div$ 1.0567 = 946.34 cu. cm.

Page 23.

1. A liter = 1 cu. dm. = 1,000 cu. cm. One cu. cm. of pure water weighs 1 gram; 1,000 cu. cm. of water weigh 1,000 grams, or 1 Kg.

2. 1,000 g. $\times$ 1.8 = 1,800 g.

3. 1,250 cu. cm. of water weigh 1,250 g.

1,250 g. $\times$ 0.8 = 1,000 g. or 1 Kg.

4. Since a liter of water weighs 1,000 g., 250 g. of water is $\frac{1}{4}$ of a liter of water.

5. 1 cu. dm. = 1,000 cu. cm. 1 cu. dm. of water, therefore, weighs 1,000 g. = 1 Kg.

6. 1 liter = 1,000 cu. cm.; 1 dl. = 100 cu. cm. 1 dl. of water weighs 100 times 1 g. = 100 g. = 1 Hg.

7. 18 cu. in. × 19 × 20 = 6,840 cu. in.
6,840 ÷ 231 = 29.6+, the number of gals.

8. 25 cu. cm. × 35 × 75 = 65,625 cu. cm.
65,625 ÷ 1,000 = 65.625, the number of liters.

NOTE. — Exercises 7 and 8 are intended to illustrate the advantage of the decimal scale as used in the metric measures. It is easier to divide by 1,000 than to divide by 231.

Page 37.

11. Be sure that the pupil understands that he is testing the stiffness and not the strength of the beams used. (*a*) The deflection is proportional to the load. (*b*) The deflection is proportional to the cube of the length. (*c*) The deflection is inversely proportional to the breadth. (*d*) The deflection is inversely proportional to the cube of the thickness, i.e., depth.

$$v \propto \frac{l^3}{bd^3}.$$

13. The elongation is proportional to the length of the wire, and inversely proportional to the cross-section of the wire, i.e., to the square of the diameter.

$$E = \frac{l}{d^2}.$$

14. The angle of torsion is proportional to the force of torsion.

15. The angle of torsion is proportional to the length of the rod.

Page 42.

4. Air held in solution is largely expelled by boiling. Fewer air bubbles will, therefore, accumulate on the inner wall of the tumbler that contains the boiled water. See the next exercise.

Page 54.

6. Porosity.

7. No. Scientifically speaking, pores are the minute interstices between molecules; intermolecular spaces.

8. Fluids include liquids. Both terms imply freedom of molecular motion, but the tendency of the molecules to cohere pertains necessarily to liquids, and not necessarily to fluids.

9. Water and steam are identical substances, varying only in respect to their physical condition (§ 41). The nature of a substance is determined by the nature of its molecules (§ 6) and a physical change, as from water to steam, does not change the molecule (§ 9). As there is no chemical change in passing from water to steam, no molecule is changed in any way; they remain equal in size.

10. A cubic inch of water makes about a cubic foot of steam. As there is no increase in the size or the number of the molecules, the increased volume must be due to an increase in the distances between them.

12. See § 45.

13. See Experiment 36.

14. See § 33 (*a*). Extension, impenetrability, weight, indestructibility, inertia, etc., pertain to all matter and are, therefore, universal properties. Hardness, malleability, ductility, etc., pertain to only certain forms of matter and are, therefore, characteristic of those that possess them.

15. See § 52 (*b*). Dialysis and evaporation.

16. Divide the number of pounds by 2.2046. See § 25.

18. $n \propto \frac{1}{l}$

19. See § 33 (*a*). Tenacity is proportional to sectional area, and that is proportional to the square of the diameter.

$$(14.19)^2 : (17.90)^2 : : 15 : x.$$

See Appendix 4. The squares of the diameters in mils may be found in the column headed circular mils, and the proportion written directly from the table, thus:

$$201.5 : 320.4 :: 15 : x.$$

20. From the table of wire dimensions, it appears that No. 27 wire has a sectional area twice that of No. 30 wire. Consequently, No. 27 iron wire has a breaking strength of 10 pounds. Comparing these brass and iron wires of the same size (No. 27), it appears that the tenacity of spring-brass is 1.5 times that of annealed iron.

Page 55.

1. Steam passes through the cardboard and condenses upon the inner surface of the upper tumbler, thus illustrating the porosity of the cardboard.

CHAPTER II.

Page 63.

1. $500 \times 500 = 250{,}000$.

2. $321.6 \times 200 = 64{,}320$.

3. $\left.\begin{matrix} 5 \times 4 = 20 \\ 10 \times 2 = 20 \end{matrix}\right\}$ Their momenta are equal.

4. Uniformly accelerated motion, like that of a falling body which is acted upon by the attraction of the earth.

5. There are 5,280 feet in a mile.

$$\frac{5{,}280 \times 15 \times 1}{1{,}320 \times 12} = 5.$$

The momentum of the ball will be 5 times that of the stone.

6. $\left.\begin{matrix} 50{,}000 \times 2 = 100{,}000 \\ 10{,}000 \times 10 = 100{,}000 \end{matrix}\right\}$ Their momenta are equal.

7. $25 \times 60 = 1{,}500$, the momentum of the first, and, consequently, of the second. $1{,}500 \div 40 = 37.5$. The velocity of the second is 37.5 ft. per second.

8. $100 \times 20 = 2{,}000$.
$2{,}000 \div 500 = 4$, the number of kilograms.

9. From a point, *C*, draw three lines $3\frac{1}{2}$ inches, 5 inches, and 6 inches respectively in length. Draw the first line toward the right hand; the second line downward; and the third line downward and toward the left so as to make an angle of 45° with the second line. Remember the old geographical rule: "The top of the map is north; the bottom, south; the right hand, east; and the left hand, west." The three arrowheads point from *C*.

10. $\frac{40 \times 5{,}280}{60 \times 60} = 58\frac{2}{3}$, the velocity in feet per second.

11. $\frac{92{,}390{,}000 \times 2}{(16 \times 60) + 36}$.

12. See Formula (3) on page 59.

$$l = \tfrac{1}{2}at^2 = 10 \text{ ft.} \times 6^2 = 360 \text{ ft.},$$

the distance traversed in 6 seconds.

See Formula (2) on page 58.

$$v = at = 20 \text{ ft.} \times 6 = 120 \text{ ft., the final velocity.}$$

13. The load of 50 Kg. $= 2.2046 \times 50 = 110.23$ lbs. (see § 25). This load is 7.35 times as great as the load that can be carried by the No. 27 wire. We must, therefore, have a wire that is 7.35 times as great in sectional area. From the table of wire dimensions, we find that the diameter of No. 27 wire is 0.01419 of an inch.

$$1 : 7.35 :: (0.01419)^2 : x^2.$$

$x = 0.03847$ of an inch, the diameter required. This is greater than the diameter of No. 19 wire and less than that of No. 18 wire. An easier solution for practical purposes is to multiply 201.5, the number of circular mils for No. 27 wire, by 7.35, the ratio between the given loads. This gives 1,481 circular mils, which is greater than that for No. 19 wire and less than that for No. 18 wire. No. 18 wire must be used.

Page 64.

1. Draw two diameters at right angles, thus dividing the circumference into arcs of 90° each. From the extremities of these diameters as centers, draw arcs cutting the circumference into arcs of 30° each. Use the protractor only for the trisection of the 30° arcs.

Page 68.

1. The locomotive must travel 360 ft. + 120 ft. = 480 ft. It moves $\frac{1}{2}$ mile or 2,640 ft. per minute, or 44 ft. per second. $480 \div 44 = 11$ nearly, the number of seconds.

2. See § 66 (*a*). $32.16 \times 25 = 804$, the number of poundals.

3. $980 \times 5{,}000 = 4{,}900{,}000$, the number of dynes.

4. 13,825 dynes. Multiply the number of grams in a pound by the number of centimeters in a foot.

5. $\frac{64 \times 16 \times 1{,}300}{1 \times 1{,}200} = 1{,}109\frac{1}{3}$. The momentum of the cannon-ball is $1{,}109\frac{1}{3}$ times that of the bullet.

6. See solution to Laboratory Exercise 11, page 37. The deflection is proportional to the cube of the length.

$$3^3 : 4^3 :: 0.5 : x.$$

7. Inertia.

Page 79.

1. Adopt any convenient scale, as 0.1 of an inch to the pound. Then the 100-pound force will be represented by a line 10 in. long, and the other force by a line 15 in. long. See § 69 (1). The resultant will be represented by a line 25 in. long.

2. See § 69 (2). Using the same scale as in Exercise 1, the resultant will be represented by a line 5 in. long, and the motion will be in the direction of the greater force.

3. Suppose we adopt the scale of an inch to the mile. Draw a horizontal line 4 inches long to represent the force of the oars. From one end of this line, draw a vertical line 3 inches long to represent the force of the current. Join the free ends of these lines; the hypotenuse thus formed will represent the resultant of these two forces.

$$3^2 + 4^2 = 25. \quad \sqrt{25} = 5.$$

See § 70 (*a*). The boat will move in the direction indicated by the hypotenuse and with a velocity of 5 miles per hour. Of course, the problem means that the boat is headed directly across the stream.

4. Draw a vertical line, 64 units (as mm., or 16ths of an inch) long. From the foot of this line draw a horizontal line

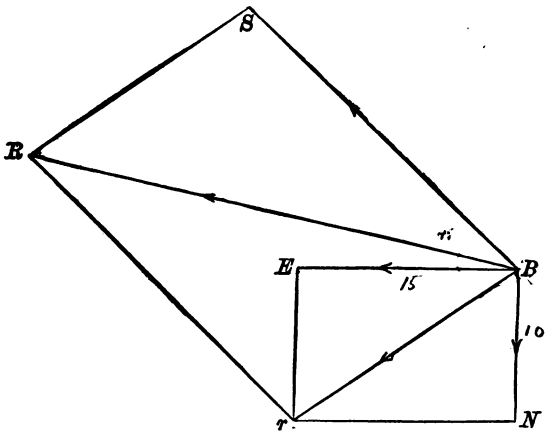
24 units long. (Use the same kind of units that you adopted for the vertical line.) Join the free ends of these lines. The hypotenuse will be the graphic representation of the resultant.

$$64^2 + 24^2 = 4{,}672. \quad \sqrt{4{,}672} = 68+.$$

5. Draw, as before, a vertical line $(3 \times 20 =)$ 60 units long, and a horizontal line $(12 \times 20 =)$ 240 units long. Draw the hypotenuse.

$$60^2 + 240^2 = 61{,}200. \quad \sqrt{61{,}200} = 247+.$$

6. Draw $BN = 10$ units of length. From B, draw BE at right angles to BN and make it 15 units long. Complete the



parallelogram and draw the partial resultant, Br. From B, draw BS at an angle of 45° from BE, and make it 25 units long. Complete the parallelogram, and draw the diagonal, BR, which will represent the complete resultant. The scale here adopted is 2 mm. per pound. The line, BR, being 67 mm. long, represents a force of $33\frac{1}{2}$ pounds. The direction of the resultant is a little to the north of west, as is indicated by the angle, EBR.

7. See § 60 (3). The momentum of the gun is equal to the momentum of the projectile. As the gun is heavier than the projectile, its velocity must be less than that of the projectile, in order that the products of the numbers representing the weight and velocity in each case may be equal.

8. The width of the river is represented by a line 4 units long; the actual course of the boat by a line 5 units long. If the 4 units represent 1 mile, the 5 units will represent $1\frac{1}{4}$ miles, the distance that the boat moves. It takes no longer. See § 62.

9. See Fig. 94, in which *LM* represents the plank, and *MN* the distance that one end of it is raised. *WC* represents the weight of the cask. From *W*, draw *WD* perpendicular to *LM*. From *C*, draw *CD* parallel to *LM*. Complete the parallelogram, *WBCD*. The force of gravity represented by *WC* may be resolved into two components, represented by *WD* and *WB*. *WB* represents the force with which the cask tends to roll down the plank. This tendency may be successfully resisted by a force represented by *WB'*, equal to *WB* and opposite in direction. $WB = \frac{1}{2}\ WC$, as may be seen by direct measurement or as may be proved geometrically, the triangles *WBC* and *LNM* being similar. Hence, the muscular force needed is 25 lbs. or more.

10. See § 66 (*a*). $32.16 \times 60 = 1{,}929.6$, the number of poundals.

11. 60 Kg. = 60,000 g.
$980 \times 60{,}000 = 58{,}800{,}000$, the number of dynes.

12. See § 65, and § 66 (*a*).
$32.16 \times 10 = 321.6$, the number of poundals.

13. $1{,}000 \div 20 = 50$, the number of grams.

14. $12 \times 6 = 72$, the number of dynes.

15. $490 \div 70 = 7$, the number of centimeters.

16. The wind strikes obliquely upon the faces of the blades and is resolved by the blades into two components, as it is in

the case of the obliquely set sails of vessels. One of these components tends to move the blade in a direction perpendicular to that of the wind. The corresponding blade on the opposite side of the wheel is similarly urged in the opposite direction, so that the force of the wind upon each such pair of blades constitutes a couple that helps to turn the wheel about its axis.

17. See § 78.

$$\text{C.F.} = \frac{mv^2}{r} = \frac{10 \times 20^2}{5} = 800.$$

18. Compute the length of the hypotenuse of an imaginary right-angled triangle with sides of 5 and 12 respectively. See § 70 (*a*).

$$5^2 + 12^2 = x^2;\ 169 = x^2;\ 13 = x.$$

The resultant has a force of 13 Kg.

19. In this case, the hypotenuse of the right-angled triangle is 30, and one of the other sides is 18.

$$30^2 - 18^2 = x^2.$$

$x = 24$, the numerical value of the component sought.

20. See Experiment 45, and § 69 (3).

$$8 + 11 = 19.$$
$$19 :\ 8 :: 57 : 24.$$
$$19 : 11 :: 57 : 33.$$

The point will be 24 inches from the line of the 11-pound force, and 33 inches from the line of the 24-pound force.

21. See § 58. The southward and the eastward motions are equal. The parallelogram is a square with a diagonal of 30 units.

$$(30)^2 \div 2 = 450.$$
$$\sqrt{450} = 21+, \text{ the number of miles.}$$

22. See § 78.

$$\text{C.F.} = \frac{mv^2}{r} = \frac{10{,}000 \times (2{,}000)^2}{1{,}000} = 40{,}000{,}000,$$ the number of dynes.

23. (*a*) See § 60 (3). The action on the sail will be balanced by the reaction on the bellows and the boat will not be moved by the operation. (*b*) The reaction on the bellows will push it and the boat ahead.

Page 81.

1. See § 69 (3).

2. From the ends of a vertical line 5 inches long, draw toward the right hand horizontal lines $3\frac{1}{2}$ and $5\frac{1}{2}$ inches long respectively. An arrowhead on each line should point to the right. From a point on a vertical line, $\frac{7}{18}$ of its length from the $5\frac{1}{2}$-inch line and $\frac{11}{18}$ of its length from the $3\frac{1}{2}$-inch line, draw a horizontal line to the left and 9 inches long. This line with an arrowhead pointing to the right represents the resultant. The same line with an arrowhead pointing to the left represents the equilibrant.

3. The line drawn downward from *O* with a length equal to *OD* represents the gravity of the unknown weight. *OD* represents the equilibrant of that weight.

4. *ZT* represents the equilibrant of the 10-pound pull of the weight. The scale adopted is $\frac{1}{2}$ inch = 1 pound. See § 70.

7. Draw an equilateral triangle as follows: lay off a line *AB*, of convenient length. From *A* as the center and with a radius equal to *AB*, describe an arc. From *B* as a center and with the same radius, describe a second arc that cuts the first arc. Mark the point of intersection of the two arcs, *C*, and join *AC* and *BC*.

8. The angle measures 127° 3′+.

Page 90.

1. See § 81. $\frac{8{,}250 \times 176}{550 \times 60 \times 4} = 11$, the number of H. P.

SUGGESTION: Reduce by cancellation.

2. See § 85. $\frac{192.96 \times 10{,}000}{64.32} = 30{,}000.$

3. The direction makes no difference.

$\frac{50 \times 19.6 \times 19.6}{2 \times 9.8} = 50 \times 19.6 = 980$, the number of kilogram-meters.

4. $50 \times 750 =$ the momenta.

$\frac{50 \times 750}{75} = 500$, the required velocity in feet.

5. The velocity is 500 m. per minute, or $8\frac{1}{3}$ m. per second.

$\frac{40 \times 8\frac{1}{3} \times 8\frac{1}{3}}{2 \times 9.8} = 141\frac{81}{441}$, the number of kilogram-meters.

6. $\frac{1,500 \times 2,376}{550 \times 60 \times 3} = 36$, the number of H. P.

7. That quantity of water weighs about (62.5 lbs. $\times$ 300 =) 18,750 pounds.

$\frac{62.5 \times 300 \times 132}{33,000} = 75$, the number of H. P. necessary.

8. A velocity of 20 miles per hour is one of $29\frac{1}{3}$ feet per second.

$\text{K. E.} = \frac{100 \times 29\frac{1}{3} \times 29\frac{1}{3}}{64.32} =$ the number of foot-pounds.

9. (*a*) $6,000 \times 50 = 300,000$, the number of foot-pounds.
(*b*) $300,000 \div 16,500 =$ the number of H. P.

10. $2 = \frac{10,000 \times 100}{33,000\, m}$. $\therefore\ m = 15\frac{5}{33}$, the number of minutes.

11. $2 = \frac{10,000\, x}{550 \times 30}$. $x = 3.3$, the number of feet.

12. It is often convenient to remember that 550 foot-pounds per second is equivalent to 33,000 foot-pounds per minute. $1,650,000 \div 33,000 = 50$, the number of H. P.

13. $\frac{2,376 \times 1,000}{33,000 \times 2} = 36$, the number of H. P.

14. K. E. $= \frac{201 \times 400}{64.32} = 1{,}250$, the number of foot-pounds that the moving sphere can perform. This working power is the exact measure of the work performed upon it.

15. A resistance of 8 pounds per ton signifies that to move a ton one foot on the rails involves as much work as to lift 8 pounds one foot high, or 8 foot-pounds. To move 10 tons 50 ft. on the rails would require 4,000 foot-pounds. The additional work done in giving kinetic energy to the car will be measured by the kinetic energy of the car. The velocity of the car is 4.4 ft. per second.

$$\text{K. E.} = \frac{20{,}000 \times 4.4 \times 4.4}{64.32} = 6{,}019+,$$

the number of foot-pounds that the car can perform, or the amount of work done in giving the car the velocity specified. $4{,}000 + 6{,}019 = 10{,}019$, the total amount of work, in foot-pounds, that was done on the car.

16. The triangle or parallelogram will not "close." None of the given forces can be the equilibrant of the other two.

17. See § 46 (*a*).

18. The velocity along the track is $29\frac{1}{3}$ ft. per second. The problem is to find the hypotenuse of a right-angled triangle, the sides of which measure respectively $29\frac{1}{3}$ and 20 ft. This magnitude of 35.5 ft. represents the velocity per second.

19. $15 \times 20 \div 4 = 75$, the number of dynes.

20. The momenta of gun and projectile $= 250 \times 1{,}420 = 355{,}000$. This divided by 34,000, the weight of the gun, gives 14.79 as the required velocity in feet.

21. $6 \times 3.14159 \div 5 = 3.769$, the velocity in feet per second. The C. F. $= \frac{20 \times (3.769)^2}{3}$. See § 78.

22. $50 \times 10 = 500$, the number of foot-pounds.

23. The force is resolved into two components, one parallel to the track and producing motion; the other perpendicular to the track and producing no motion. The first component is the only one that overcomes resistance or does work. See § 79. In the parallelogram constructed for the resolution of the force, we have 50 as the diagonal of a square, the length of one side of which is to be determined.

$$\sqrt{\frac{50^2}{2}} = 35.355, \text{ the force in pounds,}$$

$35.355 \times 10 = 353.55$, the work in poundals.

24. His push contributes nothing to the motion of the car, and so it does no work on the car. See § 79.

25. The man does not push the car or help to push it; the car pushes the man. The man does no work upon the car; the car does work upon the man. The exercise is the converse of Exercise 23.

26. (*a*) See § 85. $\frac{50 \times 30^2}{32.16 \times 2}$ foot-pounds.

(*b*) See § 86. $\frac{50 \times 30^2}{2}$ foot-poundals.

27. $\frac{1{,}056{,}000}{550 \times 60 \times 60 \times 8} = \frac{1}{15}$.

Page 92.

1. It is better to take a tenth of the number of revolutions made by the spindle while the large wheel revolves 10 times. Record this ratio on the whirling table.

2. The greater the speed, the greater the centrifugal force of the ball, and the greater the length to which the cord is stretched.

3. The mercury gets as far as possible from the center of rotation, and thus rises to the upper ends of the tubes.

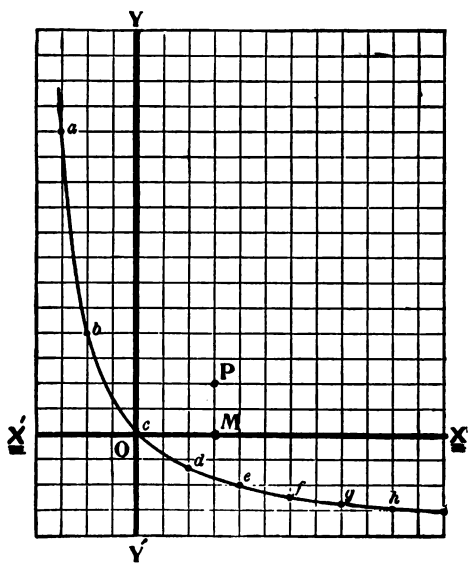
4. In the geologic-long-ago, when the earth was in a plastic condition, its rotation upon its axis would give it the spheroidal form that it now has; it would flatten out at the poles.

7. See § 69 (2). The components are equal and the resultant is zero.

8. This exercise shows that centrifugal force varies directly as the mass and inversely as the radius, as is indicated by the formula given in § 78.

10. The momenta produced by the elastic force of the cord are equal; the lighter block will move twice as fast as the other block.

11.



Page 98.

2. As the assumed distance from the earth's center is only one-fourth of the surface distance, the required weight will be only one-fourth of the surface weight. $550 \div 4 = 137\frac{1}{2}$. Or, $4{,}000 : 1{,}000 :: 550 : x$.

NOTE. — The teacher will be fortunate who finds that all of his pupils are able to handle a proportion intelligently and easily. Secure this ability if possible. Frequently vary the form of the statement from $a:b=c:d$, to $\frac{a}{b}=\frac{c}{d}$.

3. $$\frac{75^2}{50^2}=\frac{5{,}625}{2{,}500}=2\tfrac{1}{4}.$$

4. If the first and second were at equal distances from the third, the first would have, on account of its lesser mass, only $\frac{6}{9}$ or $\frac{2}{3}$ as great an attraction as the second. But being only half as far distant, its attraction is four times as great as it would be if it were at an equal distance. $\frac{2}{3}\times 4=\frac{8}{3}$. The smaller ball exerts $2\frac{2}{3}$ as much force upon the third ball as the larger one does.

$$\left(\frac{6}{9}\times\frac{50^2}{25^2}=\frac{8}{3},\text{ or } 1:x::\left\{\begin{matrix}9:6\\25^2:50^2\end{matrix}\right\},\text{ whence } x=\frac{8}{3}\right).$$

5. Reckon distances from the earth's center.

$$12{,}000^2:4{,}000^2::900:x.$$

Or, the distance from the earth's center being increased threefold, the weight will be divided by 3^2 or 9.

$$900\text{ lbs.}\div 9=100\text{ lbs.}$$

6. $1:16::4{,}000^2:x^2$. $\therefore\ x=16{,}000$, the number of miles from the earth's center. $16{,}000-4{,}000=12{,}000$, the number of miles from the earth's surface.

7. $7{,}000^2:4{,}000^2::200:x$. The man would weigh 65.3 lbs.
$7{,}000^2:4{,}000^2::100:y$. The boy would weigh 32.65 lbs.

8. $4{,}000^2:6{,}000^2::180:x$. *Ans.* 80 lbs.

9. Work as in preceding examples, or as follows:

50 lbs. $\times\ \frac{16}{25}=32$ lbs., the weight 1,000 miles above the surface.

50 lbs. $\times\ \frac{3}{4}=37\frac{1}{2}$ lbs., the weight 1,000 miles below the surface.

It would weigh $5\frac{1}{2}$ lbs. more when below the surface.

10. It would be $\frac{1}{9}$ as great in either case.

11. See solution to Exercise 23, page 92. The effective component is a force of 565.685 pounds.

$565.685 \times 12 = 6{,}788.22$, the number of foot-pounds.

12. $1{,}200 : 2{,}700 :: 4{,}000^2 : x^2$. $\therefore\ x = 6{,}000$.
$6{,}000 - 4{,}000 = 2{,}000$.

13. It would increase the weight fourfold. See § 90.

14. In such cases of mutual action between free bodies, the two bodies experience equal changes of momentum. Just before collision, the small ball had a momentum of 3,750 in a certain direction. Just after collision, it had a momentum of 1,250 in the opposite direction. One might be called positive and the other negative. The change of momentum of the small ball was 5,000. This change was produced by the reaction of the collision. The equivalent action must have given the large ball a momentum of 5,000. This divided by the weight of the large ball gives the velocity of the large ball. $5{,}000 \div 200 = 25$, the required velocity in feet per second.

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1. See §§ 97, 100.

2. See § 97.

3. See § 100 (*b*).

4. The center of mass is lower. See § 100.

5. The distance from the center of the earth being multiplied by 60, the attraction will be divided by 60^2. See § 90.

$$32.16 \text{ ft.} \div 3{,}600 = 0.0089 + \text{ ft.}$$

6. Draw two lines, respectively 10 and 7.5 cm. in length, making them include an angle of 45 degrees. Complete the parallelogram and draw the diagonal from an acute angle. Its length of 12.147 cm. represents a value of 121.47 feet per second. The direction is 25° 53′ to the east of north.

7. 5,280 ft. $\div$ 60 = 88 ft., the velocity per second. This velocity was developed in 600 seconds. 88 $\div$ 600 = 0.1466, the acceleration in feet. Force = mass $\times$ acceleration; $f = ma$. §§ 66 (*b*) and 86.

0.1466 $\times$ 100 = 14.66, the number of poundals.

8. See § 81.

$$100 = \frac{1{,}600\,x}{550 \times 60 \times 60 \times 10}; \quad x = 1{,}237{,}500.$$

Or,

1 H. P. = 550 foot-pounds in 1 second.
100 H. P. = 55,000 foot-pounds in 1 second.
= 3,300,000 foot-pounds in 1 minute.
= 198,000,000 foot-pounds in 1 hour.
= 1,980,000,000 foot-pounds in 10 hours.

Each gallon requires (8 $\times$ 200 =) 1,600 foot-pounds.
1,980,000,000 $\div$ 1,600 = 1,237,500, the number of gallons.

9. Because the base is large, and the center of mass is low.

10. Because the base is larger. See § 97.

11. One is at the center of the axle, and the other should be.

12. In the first case, the center of mass was so low that the line of direction fell within the base; in the other case, it was so high that the line of direction fell without the base.

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1. The center of mass.

2. No; it is about midway between the two sides of the cardboard.

3. The three lines should intersect at a common point.

4. The difference represents the distance that the center of mass must be raised to throw the line of direction outside the base. The products represent the amount of work that must be done in each case to overturn the box. Foot-pounds.

5. The red patch will circle around the black patch as the latter moves through the air.

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4. $l' = \frac{1}{2}g(2t-1) = 16.08$ ft. $\times 5 = 80.4$ ft.

5. $l = \frac{1}{2}gt^2 = 16.08$ ft. $\times 100 = 1{,}608$ ft.

6. $l = \frac{1}{2}gt^2 = 16.08$ ft. $\times \frac{1}{4} = 4.02$ ft.

7. $l = \frac{1}{2}gt^2 = 16.08$ ft. $\times 2.25 = 36.18$ ft.

8. 2,512 ft.

9. $l = \frac{1}{2}gt^2$; $787.92 = 16.08\,t^2$; $49 = t^2$; $7 = t$.

10. $v = gt = 32.16$ ft. $\times 7 = 225.12$ ft.

11. $v = gt = 32.16$ ft. $\times 15.5 = 498.48$ ft.

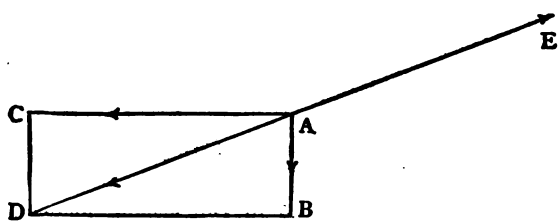
12. $7\frac{1}{2}$ oz. $+ 7\frac{1}{2}$ oz. $+ 1$ oz. $= 16$ oz., the total weight to be moved. To move this load we have the weight of the rider, a force of 1 oz. This force can give to an ounce of matter a velocity of ($g =$) 32.16 ft. per second; the same force can give to 16 oz. of matter only $\frac{1}{16}$ of this velocity.

$$32.16 \text{ ft.} \div 16 = 2.01 \text{ ft.}$$

13. $l = \frac{1}{2}gt^2$; $257.28 = 16.08\,t^2$; $4 = t$.

The ball will reach the ground at the end of 4 seconds. During that time it will move from the tower (60 ft. $\times 4 =$) 240 ft. See §§ 62 and 111.

14. Suppose the sinker to be at *A*. Gravity pulls or pushes it toward *B*. The current pushes it toward *C*. The resultant



of these tends to move it to *D*. The equilibrant of *AD* is *AE*, which represents the tension or pull of the line.

15. $v = gt;\ 80.4 = 32.16\,t;\ 2.5 = t.$

The body will rise for $2\frac{1}{2}$ seconds. At the time specified, it will have been falling $\frac{1}{2}$ second, and will have a velocity of 16.08 ft.

16. *Fl* represents the distance that gravity will move the body during the first second. *Fa* represents the velocity due to the horizontal impelling force, or the distance that force would move it in the first second. *Aa* represents the amount of deviation from a horizontal plane, as a consequence of the pull of gravity during the first second. It is equal to *Fl*. *Fc* represents the horizontal distance the projectile will move in 3 seconds. It is 3 times the initial velocity. *Dd* represents the total pull of gravity for four seconds from starting.

17. $l = \frac{1}{2}gt^2 = 16.08$ ft. $\times\ 16 = 257.28$ ft.

This is the distance the body would have moved in the given time if it had had no initial velocity. But it moved $(357.28 - 257.28 =) 100$ feet further in the 4 seconds. The initial velocity must, therefore, have been $(100 \div 4 =) 25$ feet.

18. During that time, gravity alone moved it 2,512.5 ft. The additional force moved it $(35 \times 12\frac{1}{2} =) 437.5$ ft. Together they moved it $(2{,}512.5 + 437.5 =) 2{,}950$ ft. Its final velocity due to gravity is $(32.16 \times 12.5 =) 402$ ft., to which we must add the initial velocity, making a velocity of 437 ft.

19. (*a*) $3{,}216 \div 32.16 = 100$, the number of seconds.

(*b*) The end of the 4th second of the ascent corresponds to the end of the 96th second of the descent.

$$v = gt = 32.16 \text{ ft.} \times 96 = 3{,}087.36 \text{ ft.}$$

Do not forget that this result disregards the resistance of the air; that it is true only for a freely falling (or rising) body.

(*c*) The end of the 7th second of the ascent corresponds to the end of the 93d second of the descent,

$$v = gt = 32.16 \text{ ft.} \times 93 = 2{,}990.88 \text{ ft.}$$

20. The ball has to fall from a height of 257.28 ft. Whether the force of gravity draws it vertically downward as a freely falling body, or acts with the projecting force to produce a curved path, it will reach the ground in the same time. It would fall this distance in what time?

$$l = \tfrac{1}{2} gt^2;\ 257.28 = 16.08\, t^2;\ 16 = t^2;\ 4 = t.$$

If it would take 4 seconds to fall from the highest point reached, it would require 4 seconds for it to rise to that height; it would be in the air 8 seconds. During each of these seconds it has a horizontal motion of 1,000 ft.

21. $l = \tfrac{1}{2} gt^2 = 16.08$ ft. $\times\ 25 = 402$ ft., the distance that the force of gravity would move the body in 5 seconds. The force with which it was thrown moves it 10 ft. each second, or 50 ft. during the five seconds.

$$402 \text{ ft.} + 50 \text{ ft.} = 452 \text{ ft.}$$

22.
$$v = gt = 32.16 \text{ ft.} \times 5 = 160.8 \text{ ft.}$$
$$160.8 \text{ ft.} + 10 \text{ ft.} = 170.8 \text{ ft.}$$

23. See Experiment 54. (*a*) The value of each space is 7 ft. In 5 seconds it will pass over ($t^2 =$) 25 such spaces, or 175 ft.

(*b*) Its final velocity will be ($2\,t =$) 10 times 7 ft. = 70 ft.

24. (*a*) See § 107 (*c*). The distance passed over in the first second will be half the velocity acquired during the first second. Hence, the value of the spaces traversed in this case is 10 ft. In 10 seconds it will pass over ($t^2 =$) 100 such spaces, or 1,000 ft. Or, we may say that the increment of velocity (g) under these circumstances is 20 ft., instead of 32.16 ft. Then,

$$l = \tfrac{1}{2} gt^2 = 10 \text{ ft.} \times 100 = 1{,}000 \text{ ft.}$$

(*b*) See § 107 (*a*). The ratio between the height and length of the plane is the same as that between 20 ft. and 32.16 ft.

$$20 : 32.16 :: 800 \text{ ft.} : 1{,}286.40 \text{ ft.}$$

25. (*a*) It would take just as long to rise 1,302.48 ft. as it would to fall that distance. Then,

$$l = \tfrac{1}{2} gt^2;\ 1{,}302.48 = 16.08\, t^2;\ 81 = t^2;\ 9 = t.$$

(*b*) The initial velocity of a body that can rise for 9 seconds, is the same as the final velocity of a body that has fallen for 9 seconds. Then,

$$v = gt = 32.16 \text{ ft.} \times 9 = 289.44 \text{ ft.}$$

26. During 7 seconds, gravity would give it a velocity of ($gt = 32.16$ ft. $\times 7 =$) 225.12 ft. But as its velocity is 235.12 ft., it must have had an initial velocity of (235.12 $- 225.12 =$) 10 ft. During 4 seconds, gravity would move it ($\frac{1}{2} gt^2 = 16.08$ ft. $\times 16 =$) 257.28 ft. But during each of these 4 seconds, the force of the throw moves it 10 ft. more. This amounts to 40 additional feet during the 4 seconds.

$$257.28 \text{ ft.} + 40 \text{ ft.} = 297.28 \text{ ft.}$$

27. $l = \frac{1}{2} gt^2$; $787.92 = 16.08\, t^2$; $49 = t^2$; $7 = t$.

This is the time that the second body was in falling 787.92 ft. But the first body fell 3 seconds more, or 10 seconds. During 10 seconds it would fall 16.08 ft. $\times 100 = 1{,}608$ ft.

28. During 4 seconds it will fall 257.28 ft. During 6 seconds it will fall 578.88 ft. During the 5th and 6th seconds it will fall 578.88 ft. $-$ 257.28 ft. $=$ 321.6 ft. Or we may say that its average velocity during these two seconds will be that attained at the end of the 5th second, which is 160.8 ft. Moving at this average rate for 2 seconds, it will move (160.8 ft. $\times 2 =$) 321.6 ft.

29. Inversely; as the time increases, the velocity decreases.

30. When they have been extended to an infinite distance.

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4. For the sake of illustration, suppose that the origin of co-ordinates is taken at a corner of one of the squares near the upper left-hand corner of the sheet of cross-section paper. Let each division on the axes represent 10 ft.; then the successive points will fall on successive vertical ruled lines. In

the first half second, the ball will move 10 ft. toward the right and 4.02 ft. downward. The corresponding point will be one space to the right of the axis of ordinates and about 0.4 of a space below the axis of abscissas. The co-ordinates for the successive periods will be as follows:

Seconds.	Abscissas.	Ordinates.	Seconds.	Abscissas.	Ordinates.
0.5	1	-4.02	3.0	6	-144.72
1.0	2	-16.08	3.5	7	-196.98
1.5	3	-36.18	4.0	8	-257.28
2.0	4	-64.32	4.5	9	-325.62
2.5	5	-100.50	5.0	10	-402.00

The mapping of the curve will require 11 or 12 spaces to the right of the axis of ordinates, and 49 or 50 spaces below the axis of abscissas. The same scale being used for both ordinates, the curve will not be distorted, but will accurately represent the actual path of the ball. Find the point on the curve that has an abscissa of 6.5; i.e., a point that is 6.5 spaces to the right of the axis of ordinates. This point corresponds to a period of 3.25 seconds. Measure its ordinate; i.e., the number of spaces that it is below the axis of abscissas. Remember that the scale adopted is 1 space = 10 feet, and determine the magnitude represented by the ordinate. It should be nearly 170 feet. Next, find the point on the curve that has an abscissa of 11; i.e., a point that is 11 spaces to the right of the axis of ordinates. This point corresponds to a period of 5.5 seconds. Measure its ordinate; i.e., the number of spaces that it is below the axis of abscissas. Determine the magnitude represented by this ordinate. It should be about 486 feet.

5. Divide the several values of l by the square of the corresponding number of time intervals. For instance, divide the value of l as shown in Fig. 64, by the square of 15. The several quotients will be equal to the value of l when $t = 1$.

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NOTE. — Take 39.1 inches or 99.33 centimeters as the length of a seconds pendulum.

1. The period is 3 seconds. See § 115 (3). 39.1 in. $\times 3^2 =$ 351.9 in., the length of the pendulum.

2. The period is 2 seconds. 39.1 in. $\times 2^2 = 156.4$ in.

3. We have two pendulums to compare; the seconds pendulum, the length and period of which are known, and the given pendulum, the length of which is 30 inches, but the period of which we are to find.

39.1 in. : 30 in. :: $1^2 : t^2$; $t = 0.87$. Since the period is 0.87 seconds, there will be as many oscillations in 60 seconds as 0.87 is contained times in 60, which is 68.9, the number of oscillations per minute.

4. $39.1 : 16 = 1^2 : t^2$; $t = 0.64 -$. The number of vibrations is 93.7+.

5. $60 \div \frac{1}{4} = 240$, the number of oscillations per minute.

$39.1 : l :: 1^2 : (\frac{1}{4})^2$; $l = 2.44+$ in. Or, we may say that, as the time is $\frac{1}{4}$ that of the seconds pendulum, the length will be $\frac{1}{16}$ that of the seconds pendulum, or 2.44+ in.

6. 60 sec. $\div$ 15 sec. $= 4$, the number of oscillations per minute.

$39.1 : l :: 1^2 : 15^2$. Or, since the time is 15 times that of the seconds pendulum, the length will be $(15^2 =)$ 225 times the length of the seconds pendulum.

7. $39.1 : 39.37 :: 1^2 : t^2$; $t = 1+$ sec.

Notice how closely the meter corresponds to the length of the seconds pendulum.

8. The period is 6 seconds.

$$39.1 \text{ in.} \times 6^2 = 1{,}407.6 \text{ in.} = 117.3 \text{ ft.}$$

9. 39.1 in. : 10 in. :: $1^2 : t^2$; $t = \frac{1}{2}+$. If the period is $\frac{1}{2}$ second, the number of oscillations per second is 2.

10. The period is 60 seconds. 39.1 in. $\times 60^2$ is equivalent to 11,730 ft., or more than $2\frac{1}{5}$ miles.

11. The length of this pendulum is that of the seconds pendulum. Hence the number of oscillations is 1 per second, and the period is 1 second.

12. 99.33 cm. $\times 2^2 = 397.32$ cm. $= 3.9732$ m.

13. 99.33 cm. $\times 120^2 = 1{,}430{,}352$ cm. $= 14.3+$ Km.

14. (*b*) $99.33 : 24.83 :: 1^2 : t^2$;

$$t = \sqrt{\frac{24.83}{99.33}} = \tfrac{1}{2} \text{ nearly.}$$

15. The period is $\frac{1}{8}$ second. 99.33 cm. $\times (\frac{1}{8})^2 = 1.552$ cm.

16. $99.33 : 397.32 :: 1^2 : t^2$; $t = 2$.

17. $99.33 : 11.03 :: 1^2 : t^2$; $t = \frac{1}{3}$ nearly.

18. 99.33 cm. $\times 10^2 = 9{,}933$ cm. $= 99.33$ m.

19. $99.33 : 2{,}483.25 :: 1^2 : t^2$; $t = 5$.

SUGGESTION. — Divide the 1st couplet by 99.33.

20. (*a*) 99.33 cm. $\times 4^2 = 1{,}589.28$ cm. $= 15.8928$ m.

21. See § 115 (3). $p : p' :: \sqrt{4} : \sqrt{49}$; i.e., the periods will be as 2 : 7.

22. $l : l' :: 70^2 : 80^2$; i.e., the lengths will be as 49 : 64.

23. The period of the longer pendulum will be twice that of the shorter one.

24. 39.1 in. $\times (\frac{1}{5})^2 = 1.564$ in.

25. (*a*) 39.1 in. $\times 8^2 = 2{,}502.4$ in., or 99.33 cm. $\times 8^2$
$= 6{,}357.12$ cm. $= 6{,}357+$ m.

(*b*) 39.1 in. $\times (\frac{1}{5})^2 = 0.61$ in., or 993.3 mm. $\times (\frac{1}{5})^2 = 15.5$ mm.

26. 39.1 in. $\times (3.5)^2 = 478.975$ in., nearly 40 ft.

27. Time of vibration $= 0.8$ seconds.

39.1 in. $\times (0.8)^2 = 25.02+$ in.

28. $60 : 60\frac{1}{2} :: n^2 : 400^2$; $n = 398+$.

29. See § 115.

$$1 = \pi\sqrt{\frac{l}{981}}; \quad l = 99.396, \text{ the number of centimeters.}$$

30. The period is $\frac{300}{206}$.

$$\frac{300}{206} = \frac{150}{103} = \pi\sqrt{\frac{83}{g}}; \quad \frac{150^2}{103^2} = \frac{\pi^2 \times 83}{g}; \quad g = 386\tfrac{1}{4} \text{ in.}$$

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1. $x = \frac{1}{4}$; $y = \frac{1}{9}$; $z = \frac{1}{16}$; $\frac{1}{25}$; $\frac{1}{36}$.

2. Substitute the known values in the formula given in § 115. $t = 1$, and l is determined by measurement.

3. The period is 0.5 of a second, or half as long as that of the other pendulum. Its length should be one-fourth as great.

4. The period is $\frac{1}{3}$ of a second.

7. The upper bullets correspond to shorter pendulums, and tend to move faster than the lower bullets.

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1. See § 124.

(*a*) $\frac{50 \times 15}{250} = 3$, the number of feet.

(*b*) $\frac{50 \times 15}{75} = 10$, the number of pounds.

2. (*a*) $\frac{100 \times 10}{125} = 8$, the number of pounds.

(*b*) $\frac{100 \times 10}{200} = 5$, the number of feet per second.

3. The lever has equal arms; i.e., it is a balance. The power and load are equal.

4. (*a*) 15 feet. (*b*) 10 feet. (*c*) Such a lever cannot be of the 3d class, for in such levers the weight-arm is always greater than the power-arm.

5. (*a*) First and second classes. (*b*) If of the 1st class, 30 lbs.; if of the 2d class, 40 lbs.

6. Figure the lever. It must be of the 1st class, with arms of 4 and 6 feet. See § 131.

$$40 \times 4 = 160;\ 1{,}000 \times 2 = 2{,}000;\ 160 + 2{,}000 = 2{,}160,$$

the sum of the moments for the short, and, consequently, the moment of the force to be applied at the end of the long arm to produce equilibrium. But in the case of the long arm, 6, the length of the arm, is one factor of this 2,160; therefore (2,160 ÷ 6 =) 360, the number of pounds, is the other factor. Or, more briefly,

$$6x = 2{,}160;\quad x = 360.$$

7. The lever is of the first kind, with arms of 8 and 12 feet respectively. Figure the lever. Represent the force at the end of the short arm by x. Then will the force at the end of the other arm be represented by $1{,}200 - x$. Placing the moments of the two forces equal to each other, we have,

$$8x = 12(1{,}200 - x) = 14{,}400 - 12x.$$
$$\therefore\ 20x = 14{,}400;\ x = 720;\ 1{,}200 - x = 480.$$

Ans. 720 lbs. at the end of the short arm, and 480 lbs. at the end of the long arm.

$$\textit{Proof.}\ \begin{cases} 720 + 480 = 1{,}200. \\ 720 \times \quad 8 = \ 480 \times 12. \end{cases}$$

8. Figure the lever, with the first three forces acting.

$$\begin{array}{r|l} 40 & 100 \\ 300 & \\ \hline 340 = 100 + 240. & \end{array}$$

We thus see that an additional moment of 240 is needed with the 100, to produce equilibrium. Of this 240, one factor is 80; then the other factor is 3, the number of feet from the fulcrum, on the same side as the force of 100 lbs. Or, we may

represent the four forces acting upon the lever, calling the distance of the fourth from the fulcrum, x. Then we shall have

$$\begin{array}{r|l} 40 & 100 \\ 300 & 80\,x \end{array}$$

$$340 = 100 + 80\,x; \quad x = 3.$$

SUGGESTION. — In a case like this, simple inspection will generally show on which arm to figure the fourth force as acting. If it is erroneously represented, either as to position or direction, the result will be a negative quantity. For instance, suppose that in this case we represent the force of 80 lbs. as acting on the same arm as the forces of 10 and of 100 lbs., and in a downward direction. We should then have

$$\begin{array}{r|l} 40 & 100 \\ 300 & \\ 80\,x & \end{array}$$

$$340 + 80\,x = 100; \quad 80\,x = -240; \quad x = -3.$$

Here the 3 would indicate that the force was acting at a distance of 3 ft. from the fulcrum; the *minus* sign would indicate that the force was tending to turn the lever in a direction opposite to that assumed. This contrary tendency might result from an *upward* force of 80 lbs. acting 3 ft. from the fulcrum and on the same arm as the 10 lbs., or from a *downward* force of 80 lbs. acting 3 ft. from the fulcrum and on the same arm as the 100 lbs. Due regard for the algebraic sign of the result will correct any error of this kind. In this problem, a *downward* force is specified; the problem is therefore definite.

9. Draw a line, ab, to represent, on any convenient scale, as 1 inch to the foot, a lever $6\frac{1}{4}$ feet long. Then this line will be $6\frac{1}{4}$ inches long. Measure on the line $\frac{3}{4}$ of an inch from b, for the position of the fulcrum, c. The lever is evidently of the 1st class, with arms of $5\frac{1}{2}$ feet and $\frac{3}{4}$ feet respectively. Represent a downward force of 60 pounds at a (Fig. 76). Locate d, $2\frac{3}{4}$ inches (feet) from c. Represent a downward force of 75 pounds acting at d. Represent a downward force of x pounds at b.

$$\begin{array}{r|l} 5\frac{1}{2} \times 60 = 330 & \\ 2\frac{3}{4} \times 75 = 206\frac{1}{4} & \frac{3}{4}\,x \end{array}$$

$$536\tfrac{1}{4} = \tfrac{3}{4}\,x; \quad x = 715.$$

10. Figure the lever as in Exercise 9. Place the moments tending to move a upward in one column, and those tending to move it downward in the other column.

$$\begin{array}{r|l} 40 \times 10 = 400 & 364 = 56 \times 6\frac{1}{2} \\ 28\,x & 288 = 96 \times 3 \\ \hline 28\,x + 400 = 652 & \\ 28\,x = 252 & \\ x = \quad 9 & \end{array}$$

The force of 28 lbs. must act at a distance of 9 ft. from the fulcrum. It must, therefore, act on the long arm, as the short arm is only 3 ft. long. We assumed that the force would tend to draw a downward. This assumption was not contradicted by a negative result. The force will therefore act downward, at a distance of 1 ft. from a.

11. Draw a line 18 cm. long to represent the beam. (Scale: 1 cm. = 1 ft.) Letter its extremities a and b. At a point 3 cm. from a, represent a downward force of 2,000 lbs. At a point 8 cm. from b, represent a downward force of 1,400 lbs. Consider either end as the power, and the other end as the fulcrum.

WITH POWER AT b.

$$\begin{array}{r|l} 2{,}000 \times \ 3 = \ 6{,}000 & \\ 1{,}400 \times 10 = 14{,}000 & 18\,x \\ \hline 20{,}000 \ = 18\,x & \\ 1{,}111\frac{1}{9} = \quad x & \end{array}$$

Pressure at b = $1{,}111\frac{1}{9}$ lbs.

Pressure at a = $(3{,}400 - 1{,}111\frac{1}{9} =)\ 2{,}288\frac{8}{9}$ lbs.

WITH POWER AT a.

$$\begin{array}{r|l} & 11{,}200 = 1{,}400 \times \ 8 \\ 18\,x & 30{,}000 = 2{,}000 \times 15 \\ \hline 18\,x = 41{,}200 & \\ x = \ 2{,}288\frac{8}{9} & \end{array}$$

Pressure at a = $2{,}288\frac{8}{9}$ lbs.

Pressure at b = $(3{,}400 - 2{,}288\frac{8}{9} =)\ 1{,}111\frac{1}{9}$ lbs.

12. Represent the power-arm by x and the weight-arm by $3 - x$. Call 40 the power, and 200 the weight.

$$P : W :: \overline{WF} : \overline{PF}, \text{ or } 40 : 200 :: 3 - x : x.$$
$$\therefore\ 40x = 600 - 200x;\ 240x = 600;$$
$$x = 2\tfrac{1}{2};\ 3 - x = \tfrac{1}{2}.$$

The fulcrum must be $2\frac{1}{2}$ ft. from the 40 lbs., and 6 in. from the 200 lbs.

Or, we may proceed as follows: The weight is 5 times the power; consequently, the power-arm must be 5 times the weight-arm. Dividing 36 inches, the length of the lever, into two parts, so that one shall be 5 times as great as the other, we have 30 inches and 6 inches.

13. See § 132 (*a*). Represent the *true* weight by x, and remember that 1 lb. 9 oz. = 25 oz., and that 2 lbs. 4 oz. = 36 oz.

$$25 : x :: x : 36;\ x^2 = 900;\ x = 30.$$

The true weight is 30 oz., or 1 lb. 14 oz.

14. A ought to carry 200 lbs.; B, 100 lbs. Call the 6 ft. bar a lever of the second class, fulcrum at either end, as A's end. Then the weight is 300 lbs.; the power, 100 lbs.; the power-arm, 6 ft.; the weight-arm $= x$.

$$P : W :: \overline{WF} : \overline{PF};\ 100 : 300 :: x : 6;$$
$$300x = 600;\ x = 2.$$

The weight should be hung two feet from the fulcrum; or two feet from A, and four feet from B.

15. As the beef weighs 450 pounds, A should carry 250 pounds, and B, 200 pounds. Suppose B to act as fulcrum, and A as power.

$$P : W :: \overline{WF} : \overline{PF};\ 250 : 450 :: x : 8;$$
$$9x = 40;\ x = 4\tfrac{4}{9}.$$

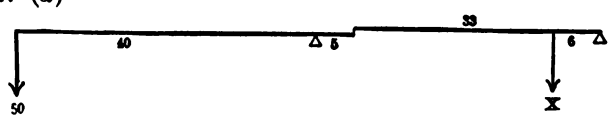
The beef should hang $4\frac{4}{9}$ ft. from the fulcrum; or $4\frac{4}{9}$ ft. from B, and $3\frac{5}{9}$ ft. from A.

16. The lever is of the 1st class, with arms of $3\frac{1}{2}$ ft. and $(16 - 3\frac{1}{2} =) 12\frac{1}{2}$ ft. respectively.

$$100 \times 12\tfrac{1}{2} = 1{,}250 = 3\tfrac{1}{2}x; \quad x = 357\tfrac{1}{7}.$$

It would require a power of $357\frac{1}{7}$ lbs. to balance the 100 lbs. On the supposition that the machine is free from friction and other hindrances to motion, anything more than $357\frac{1}{7}$ lbs. would move the load. In practice, an allowance, determined by experience, is made for these necessary hindrances.

17. (*a*)



(*b*) $50 \times 40 \times 33 = x \times 6 \times 5$; $x = 2{,}200$, the number of pounds.

(*c*) $x \times 40 \times 33 = 4{,}400 \times 6 \times 5$; $x = 100$, the number of Kg.

SUGGESTION. —Theoretically, the parts of a lever or other simple machine have no weight, suffer no loss by friction, from the stiffness of ropes, the resistance of the air, or similar causes. In practical mechanics, these have to be taken into consideration.

18. The weight of the bar, 4 lbs., may be considered as concentrated at the center of mass, 5 in. from either end. See § 94 (*a*). Let x represent the distance of the fulcrum from the end at which the 6-lb. weight is hung.

$$6x = 4(5 - x); \quad x = 2.$$

Figure the lever, and notice the equality of the moments; $3 \times 4 = 2 \times 6$.

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2. The reading of the dynamometer includes the weight of the bar.

10. In every case, the weight of the rod acts as if it was concentrated at the same point, viz., at the center of mass.

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1. $P : W :: d : D$; $x : 180 :: 6 : 36$; $x = 30$.

Any power greater than 30 lbs. will move the rudder.

2. $x : 2{,}000 :: 8 : 80$; $x = 200$.

$200 \div 4 = 50$ lbs. *Ans.* 50+ lbs.

3. See Fig. 85. $P : W :: d : D$; $x : 1{,}100 :: 1 : 10$; $x = 110$; $110 \div 4 = 27\frac{1}{2}$, the number of pounds.

4. The circumference of the axle is $\frac{1}{8}$ of the circumference of the wheel; $\frac{1}{8}$ of 85 lbs. is $14\frac{1}{8}$ lbs.

5. $70 : 300 :: x : 120$; $x = 28$.

Or, the power is $\frac{7}{80}$ of the weight; hence, the diameter of the axle must be $\frac{7}{80}$ of 120 inches = 28 inches.

6. The diameter of the circle described by the power is 36 inches.

(*a*) $x : 62.5 :: 10 : 36$; $x = 17.36+$, the number of pounds.

(*b*) 20 liters of water weigh 20 kilograms.

$x : 20 :: 10 : 36$; $x = 5.55$, the number of Kg.

7. We have two sets of two men each. The first set move in a circle 14 feet in diameter; the second set in a circle 10 feet in diameter. Find the effect produced by each set and add.

1st set	$60 : x :: 14 : 12 \times 14$; $x =$	720
2d set	$80 : x' :: 14 : 12 \times 10$; $x' =$	$685\frac{5}{7}$
	Total effect =	1,405 lbs.

8. The diameter of the circle traversed by the horse is 14 feet. The diameter of the capstan barrel is 14 inches; i.e., the first diameter is 12 times the second diameter. Hence, the circumference of the circle traversed by the horse is 12 times as great as the circumference of the barrel of the capstan. From this it follows that the horse must move 12 times as far as the house. (This may be made clear by computing the two

circumferences; one of them will show the distance the horse travels in turning the barrel around once; the other will show how much rope was wound up, or how far the house moved for that revolution of the capstan barrel.) Then the horse must travel (5 miles $\times$ 12 =) 60 miles. At the given rate, this will require (60 $\div$ $2\frac{1}{2}$ =) 24 hours.

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1. 50 pounds. The fixed pulley gives no mechanical advantage. See § 139.

2. It may be 25 lbs. (see Fig. 88), or $16\frac{2}{3}$ lbs. (see Fig. 89).

3. 75 lbs. $\times$ 7 = 525 lbs. The friction, etc., help to support the load, so that the power may be considered as 90 lbs. 90 lbs. $\times$ 7 = 630 lbs.

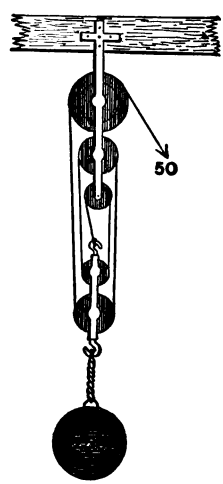
4. 2,000 $\div$ 10 = 200, the number of pounds.

5. 2,000 $\div$ 11 = $181\frac{9}{11}$+, the number of pounds. Friction resists the motion, and may be considered as added to the load. 2,500 lbs. $\div$ 11 = $227\frac{3}{11}$, the number of pounds.

6. There may be 12 cords or 13 cords. The friction will support $\frac{1}{4}$ of the weight, leaving 1,350 lbs. to be otherwise supported.

1,350 lbs. $\div$ 13 = $103\frac{11}{13}$ lbs. The power will move 13 times as fast as the weight, if either moves at all.

7.



8. The center of mass of the 125-pound board is 10 feet from each end of the board. When the boy lifts at one end of the board, the other end becomes the fulcrum of a lever of the

second class, and the power required is only half the load to be raised. He is, therefore, strong enough to lift one end of the plank as required.

$$20 : 5 :: 196 : x;\ x = 49.$$

9. It involves just as much work and requires as much energy to move it up the inclined plane as it would to lift it outright, namely, 600 foot-pounds. This is involved in the very definitions of work and energy, but it may be well to illustrate it. The power required to support the weight on the plane (ignoring friction) is determined according to the law given in § 141 (1):

$$10 \text{ ft.} : 6 \text{ ft.} = 100 \text{ lbs.} : 60 \text{ lbs.}$$

But this power of 60 pounds must act along the whole length of the plank, i.e., through a distance of 10 feet, in order to lift the weight the required distance.

$$60 \times 10 = 100 \times 6.$$

The result is 600 foot-pounds, whether it is reached in one way or in the other. But the use of the inclined plane introduces friction and increases the work to be done by one-fifth, or 120 foot-pounds, making a total of 720 foot-pounds.

10. $4{,}000 \text{ lbs.} \times \frac{3}{10} = 1{,}200 \text{ lbs.}$; or $P : W :: h : b$;

$$P : 4{,}000 :: 3 : 10;\ P = 1{,}200.$$

11. (*a*) If the power acts horizontally,

$$50 \text{ lbs.} \times \tfrac{10}{4} = 125 \text{ lbs.}\quad \textit{Ans.}\ \text{Nearly } 125 \text{ lbs.}$$

(*b*) If the power acts in the direction of the plane, first find the length of the plane:

$$\sqrt{10^2 + 4^2} = 10.77+.$$

Then, $P : W :: h : l;\ W = \text{nearly } 134.625 \text{ lbs.}$

12. $\frac{1}{8}$ of 40 lbs. is 5 lbs.

13. The power is $(\frac{25}{800} =)\ \frac{1}{32}$ of the weight. Consequently, the base is 32 times as long as the height, or 256 ft. The

length is the hypotenuse of a right-angled triangle, having sides of 8 and 256 ft. respectively.

$$\sqrt{8^2 + 256^2} = 256.1.$$

SUGGESTION. — If you have pupils in the class who can do original work, let them try to solve the problem by construction.

14. 75 lbs. : 4,000 lbs. = 1 yd. : $53\frac{1}{3}$ yds., or
75 lbs. : 4,000 lbs. = 3 ft. : 160 ft.

15. $P : W :: h : l$; $250 : 1{,}500 :: 5 : l$; $l = 30$ ft. or more.

16. The load is 400 lbs. + 10 lbs. = 410 lbs. The movable pulley divides the rope into 4 parts, and the load is, therefore, $\frac{410}{4} = 102\frac{1}{2}$ lbs. The support of the upper block has to carry both blocks, the power, and the load, or 20 lbs. + $102\frac{1}{2}$ lbs. + 400 lbs. = $522\frac{1}{2}$ lbs.

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1. 8 ft. = 96 in. 96 in. ÷ $\frac{1}{3}$ in. = 288. The power moves 96 in. while the weight moves $\frac{1}{3}$ in., or it moves 288 times as fast. Hence, the theoretical pressure will be 288 times 15 lbs. or 4,320 lbs. Deducting for friction, we have 4,320 lbs. − 240 lbs. = 4,080 lbs., the actual pressure.

2. Diameter = 16 in.; circumference = 16 in. × 3.1416 = 50.2656 in.

50 lbs. × 50.2656 × 11 =	27,646.08 lbs.
Deduct $\frac{1}{8}$ of this for friction,	3,455.76 lbs.
Available power	24,190.32 lbs.

Any resistance less than 24,190.32 lbs. may be overcome. The friction may be provided for by deducting $\frac{1}{8}$ of the 50 lbs. before multiplying by 50.2656 × 11. The problem may also be solved as follows:

$$W \times \tfrac{1}{11} = 50 \text{ lbs.} \times 50 \times 50.2656. \quad \text{See § 124 (1).}$$

SUGGESTION. — The distances here used are those traversed during one revolution of the screw.

3. $$\frac{4\frac{1}{2} \times 2 \times 12 \times 3.1416}{1\frac{1}{4}} = 271.4+$$

Of course this ignores friction.

4. 15 lbs. $\times$ 8 = 120 lbs.

$P : W :: r : R$; $\quad 25 : W :: 4\frac{1}{2} : 30$; $\quad W = 166\frac{2}{3}$.

The boy could support a load of $166\frac{2}{3}$ lbs. with the wheel and axle, without the aid of the inclined plane. With the inclined plane, he could support 20 times as great a load, or $3{,}333\frac{1}{3}$ lbs. He could lift anything less than $3{,}333\frac{1}{3}$ lbs.

6. $$\frac{500 \text{ lbs.} \times 7 \times 2 \times 12 \times 6}{11} = 45{,}818.18 \text{ lbs.}$$

Deducting $\frac{1}{4}$ of this for loss by friction .	11,454.54 lbs.
we have, for the force exerted	34,363.64 lbs.

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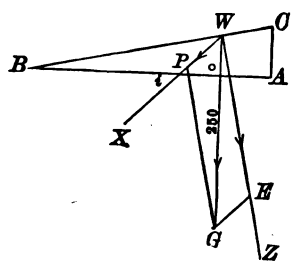
3. $P : W :: 1 : 4$.

4. There is no difference in the work performed. The power is $\frac{1}{4}$ as large, but it moves 4 times as far.

5. The ratio is 16. In general terms, $P : W :: 1 : 2^n$, or $W = P \times 2^n$, in which n represents the number of movable pulleys.

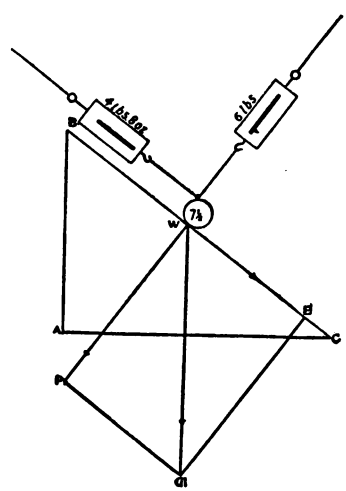
6. Construct the figure very carefully. The larger it is, the better. Represent the base, AB, by a line at least 20 cm. long, and make AC one-fifth as long. Of course, AB is supposed to be horizontal, so that the angle B measures the inclination of the plane with reference to the horizon. Anywhere on BC, take the point W to represent the position of the globe. Represent the gravity of the globe by a vertical line. If you adopt the scale of 1 mm. = 2 Kg., this line, WG, will be 12.5 cm. long. From W,



draw *WX*, so that it shall make an angle of 45° with *AB*. This may be done by taking $oi = oW$, and drawing a line from *W* through *i*. From *W*, draw *WZ* perpendicular to *BC*. From *G*, draw *GP* parallel to *WZ*, and *GE* parallel to *WP*. *WE* represents the pressure of the globe upon the plane, and *WP* the tendency to move in the direction *WX*. This tendency must be resisted by the supporting force which equals it in intensity and is opposite to it in direction. *PW* will measure 30+ mm., and represent a force of 60+ Kg.

7. Draw *ABC*, *WG*, and *WZ*, as in the last problem. From *W*, draw *WX'* perpendicular to *WX*. From *G*, draw a line parallel to *WZ*, and prolong it until it intersects *WX'* at *P'*. Complete the parallelogram by drawing *GE'* parallel to *WP'*. Measure *GP'*, and find its value according to the scale adopted.

SUGGESTION. — In the 6th problem, the supporting force tends to lift the globe from the plane, and thus to make its pressure on the plane less than its weight; in the 7th, the supporting force tends to press the globe against the plane, and thus to make its pressure on the plane greater than its weight.



9. Draw the right-angled triangle, *ABC*, with its sides as 3 : 4 : 5, as shown in the accompanying figure. Then will *AC* represent the base, and *BC* the length of the plane. From *W*, the point where the ball rests on the plane, draw the vertical line, *WG*, 7½ units long to represent the weight of the ball. From *W*, draw a line perpendicular to *BC*, and from *G* draw a line parallel to *BC*, so that they will intersect at *P*. Complete the parallelogram *WPGE*. *WE* represents

the tendency of the ball to roll down the plane, and *WP* represents the pressure of the ball on the plane. As the triangle, *WPG*, is similar to the triangle *BAC*, it follows that

$$PG : PW : WG :: 3 : 4 : 5;$$
$$PG : WG :: 3 : 5;\quad PG : 7\tfrac{1}{2} :: 3 : 5;\quad PG = 4\tfrac{1}{2}.$$
$$PW : WG :: 4 : 5;\quad PW : 7\tfrac{1}{2} :: 4 : 5;\quad PW = 6.$$

But *PG* (or its equal, *WE*) represents the tendency of the ball to roll down the plane, and its value should be indicated by the reading of the dynamometer that resists that tendency. Similarly, *PW* represents the pressure of the ball on the plane, and its value should be indicated by the reading of the dynamometer that measures that pressure.

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1. See § 151 (2). The imaginary column has a base of 30×20 and an altitude of 10 ft. Cubic contents = 6,000 cu. ft. The weight of 6,000 cu. ft. of water is 62.42 lbs. × 6,000 = 374,520 lbs.

SUGGESTION. — Allow the pupil to use 62½ lbs. as the weight of a cu. ft. of water, if he prefers to do so. That value is more easily remembered, and nearly enough accurate for non-professional purposes.

2. Bulk of imaginary column = $(6 \times 10 \times 3 =)$ 180 cu. m. = 180 Kl. = 180,000 liters. § 21. Each liter of water weighs 1 Kg. Hence the pressure is 180,000 Kg.

3. $5 \times 12 \times 6 = 360$, the number of cu. ft.
62.42 lbs. × 360 = 22,471.2 lbs.

4. $2 \times 4 \times 2 = 16$, the number of cu. m.
16 cu. m. = 16,000 l., which weigh 16,000 Kg.

5. 2 cu. yds. = 54 cu. ft.
62.42 lbs. × 54 = 3,370.68 lbs.

6. 2 cu. m. = 2,000 l.
2,000 l. of water weigh 2,000 Kg.

7. 25 cu. ft. of water weigh (62.42 lbs. × 25 =) 1,560.5 lbs.

8. $30 \times 30 \times 800 = 720{,}000$, the number of cu. cm. This quantity of water weighs 720,000 g. or 720 Kg.

9. $3 \times 3 \times 7 = 63$, the number of cu. ft.
62.42 lbs. $\times$ 63 = 3,932.46 lbs.

10. The dimensions of the vessel are $12 \times 12 \times 12$. The acid stands 8 in. deep. $12 \times 8 \times 4 = 384$, the number of cu. in. in the imaginary column. This is $\frac{2}{9}$ of a cu. ft. If the imaginary column were of water, it would weigh $\frac{2}{9}$ of 62.42 lbs. = 13.871+ lbs. Such a column of acid would weigh 1.8 times 13.871+ lbs. = 24.9678+ lbs.

11. $237 \times 35 = 8{,}295$, the number of cu. cm., and consequently the number of grams.

12. $237 \times 35 = 8{,}295$, the number of cu. in.
62.42 lbs. $\times \frac{8}{17}\frac{2}{29}\frac{5}{28} = 299.7$ lbs.

13. The water must stand $(\frac{27}{6} =)$ $4\frac{1}{2}$ ft. deep. The sides subjected to lateral pressure have an area of $(10 \times 4\frac{1}{2} =)$ 45 sq. ft. $45 \times 2\frac{1}{4} = 101\frac{1}{4}$, the number of cu. ft. in the imaginary column producing lateral pressure. There are 27 cu. ft. $(2 \times 3 \times 4\frac{1}{2})$ in the column producing pressure on the bottom. $101.25 + 27 = 128.25$.

62.42 lbs. $\times$ 128.25 = 7,999.365 lbs.

14. If the lever of the press is of the second class, as shown in Fig. 108, the piston of the pump will go down with a force of (75 lbs. $\times$ 6 =) 450 lbs. The area of the cylinder being 200 times as great as that of the piston, the weight will be 200 times 450 lbs., or 45 tons. If the lever is of the first class, the weight will be (75 lbs. $\times$ 5 $\times$ 200 =) 75,000 lbs. = $37\frac{1}{2}$ tons.

15. $2 \times 3 \times 5 = 30$, the number of cu. ft.
62.42 lbs. $\times$ 30 = 1,872.6 lbs.

16. See Appendix.
Area $= \pi R^2 = 3.1416 \times 100 = 314.16$.
314.16 $\times$ 48 = 15,079.68, the number of cu. in.
15,079.68 cu. in. = 8.72 cu. ft.
62.42 lbs. $\times$ 8.72 = 544.3+ lbs.

17. 1,000 cm., or 10 m.

18. Draw a sectional view of the vessel and tube. The water in the tube is 35 cm. higher than the upper end of the cylinder; the pressure is 35 grams per square centimeter. The water in the tube is 65 cm. higher than the lower end of the cylinder; the pressure is 65 grams per square centimeter.

19. Draw a sectional view.

(*a*) The jar contains 2,000 cu. cm., and the tube 30 cu. cm. The 2,030 cu. cm. of water weigh 2,030 grams.

(*b*) The base is 50 cm. below the surface of the water, and the pressure is 50 grams per square centimeter.

(*c* and *d*) The pressure on the base is uniform.

20. (*a*) The surface described is 30 cm. below the surface of the water, and the pressure is 30 grams per square centimeter.

(*b*) The pressure on the cover is uniform.

(*c*) $99 \times 30 = 2{,}970$, the number of cubic centimeters in the imaginary column, and the number of grams pressure.

21. The pressure on the base is $(100 \times 50 =)$ 5,000 grams.
The pressure on the cover is $(99 \times 30 =)$ 2,970 grams.
The weight of water is 2,030 grams.

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3. The difference in the mercury levels measures the upward pressures at the several depths of the water levels in the tube below the water levels outside the tube. The upward pressure varies as the depth.

4. If the line is straight, it shows that m and w vary alike, i.e., in the same ratio. If the line is curved, it shows that one varies more rapidly than the other.

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1. It will lose the weight of 1 cu. dm. of water. § 153. This is the weight of a liter of water, 1,000 g. or 1 Kg.

2. It would lose the weight of 1 cu. dm., or 1 liter of the liquid (mercury), which would weigh 13.6 times 1 Kg., or 13.6 Kg. = 13,600 g.

3. The iron is supposed to be *immersed* in the mercury. The mercury pushes it up with a force of 13,600 g. Gravity pulls it down with a force of 7,780 g. It must be held down by an additional force of 5,820 g. It can carry a load of 5,820 g. If the iron is allowed *to float* on the mercury, it will displace only 7,780 g. of mercury, thus losing its own weight. § 154.

4. The weight of a cu. ft. of water, or 62.42 lbs.

5. It will displace 100 cu. cm. of water. It will therefore lose 100 g. in weight. The remaining weight will be 1,035 g.

6. It loses 10 g. in weight when placed in water. It therefore displaces 10 g. or 10 cu. cm. of water; its own bulk is 10 cu. cm.

7. Draw a sectional view.

(*a*) The cube contains ($20 \times 20 \times 20 =$) 8,000 cu. cm.
The tube contains ($2 \times 2 \times 10 =$) 40 cu. cm.
The 8,040 cu. cm. of water weigh 8,040 g.

(*b*) The imaginary column (see § 151) contains ($20 \times 20 \times 30 =$) 12,000 cu. cm., and weighs 12,000 g. The downward pressure is 12,000 g.

(*c*) The top of the vessel that is subjected to an upward pressure is ($400 - 4 =$) 396 sq. cm. The imaginary column measures 3,960 cu. cm. The upward pressure is 3,960 g. The downward pressure minus the upward pressure represents the net downward tendency or weight. 12,000 g. − 3,960 g. = 8,040 g.

8. In Fig. 117, the weight of the particle is represented by *BW*. This force is resolved into two components, of which *BE* is perpendicular to the surface of the water. This component is met by the resistance of the water. The other component, *BD*, pulls the water particle to a lower level. When the surface becomes level, *BE* coincides with *BW*, and *BD* vanishes.

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1. As it displaces only half of its own volume of water, it weighs only half as much as its own volume of water (see § 154). Consequently, its density is one-half, or 0.5.

2. The volume of a sphere $= \frac{1}{6}\pi D^3$ (see Appendix 1) = 8,181.2 cu. cm. This quantity of water would weigh 8,181.2 g. This quantity of aluminium would weigh 2.6 times as much, or 21,271.12 g. = 21.27+ Kg.

3. 52.35 g. ÷ 5 g. = 10.47, the density of silver.

4. (*a*) 695 g. ÷ 83 g. = 8.37+, the density of brass.
(*b*) 83 g. × 0.792 = 65.736 g.
695 g. − 65.736 g. = 629.264 g.

5. 708 gr. ÷ 1,000 gr. = 0.708, the density of the benzoline.

6. 2.4554 oz. − 2.0778 oz. = 0.3776 oz.
2.4554 oz. ÷ 0.3776 oz. = 6.5+.

7. 4.6764 oz. ÷ 0.2447 oz. = 19.11, the density of the gold.

8. See § 160 (2). 970 gr. − 895 gr. = 75 gr.
970 gr. − 910 gr. = 60 gr.
60 gr. ÷ 75 gr. = 0.8.

9. 23 gr. ÷ 25 gr. = 0.92, the density of the oil.
19 gr. ÷ 25 gr. = 0.76, the density of the alcohol.

10. 1,536 g. − 1,283 g. = 253 g.
1,536 g. ÷ 253 g. = 6.07.

11. Subtract the weight of bottle from the other two weights. 4.2544 g. ÷ 4.1417 g. = 1.027+.

12. (1) Weight of wood and sinker in air, 14 g.
(2) " " " " water, 8.5 g.
(3) " " water displaced by both, 5.5 g.
(4) " " " " sinker
(10 g. ÷ 10.5 =) . . . . . .954 g.
(5) Weight of water displaced by wood, 4.546 g.
(6) Density of the wood (4 g. ÷ 4.546 g. =) 0.897.

13. Density of the metal is 1.73; of the unknown liquid, 0.67.

14. 2,160 gr. − 1,511.5 gr. = 648.5 gr.
2,160 gr. ÷ 648.5 gr. = 3.33+.

15. (1) Weight of ice and lead in air . . 24 lbs.
(2) " " " " water . 13.712 lbs.
(3) " " water displaced by ice and lead . . . . . 10.288 lbs.
(4) Weight of water displaced by lead, 1.4 lbs.
(5) " " " " ice . 8.888 lbs.
(6) Density of ice (8 lbs. ÷ 8.888 lbs. =) 0.9+.

16. (*a*) 600 g. − 545 g. = 55 g.
600 g. ÷ 55 g. = 10.9+.
(*b*) 600 g. − 557 g. = 43 g.
43 g. ÷ 55 g. = 0.78+.
(*c*) 600 cu. cm. ÷ 10.9 = 55+ cu. cm.

17. The simple question is, "How much does that stone weigh in water?"

§ 156 $2.5 = \frac{300}{300 - w'}$. ∴ $w' = 180$ lbs.

18. An equal bulk of water weighs 1,000 g.

870 g. ÷ 1,000 g. = 0.87, the density of the turpentine.

19. The volume of the fragments was 1,000 cu. cm. − 675 cu. cm. = 325 cu. cm. The fragments weighed 1,487.5 g. − 675 g. = 812.5 g. An equal bulk of water would weigh 325 g.

812.5 g. ÷ 325 g. = 2.5, the density of the mineral.

20. The 800 cu. cm. of water weigh 800 g. The 200 cu. cm. of sand weigh 1,350 g. − 800 g. = 550 g. An equal bulk of water weighs 200 g. 550 g. ÷ 200 g. = 2.75, the density of the sand.

21. $\frac{1{,}000 - 40}{120} = 8$, the density of the brass. § 157.

22. $\frac{2{,}000 + 3{,}400}{2{,}000 + 1{,}000} = 1.8$, the density of the acid. See § 160 (3).

23. The given body weighs 10 g. It displaces 2.5 g. of water (an equal bulk of water weighs 2.5 g.).

10 g. ÷ 2.5 g. = 4, the density.

24. The coal would weigh 2.4 times as much as a cubic foot of water or (62.5 lbs. × 2.4 =) 150 lbs. It would displace 1 cu. ft. of the solution, which would weigh (62.5 lbs. × 1.2 =) 75 lbs. It will lose 75 lbs. weight when in the saline solution. 150 lbs. − 75 lbs. = 75 lbs.

Or, we may say that the coal will weigh as much as 2.4 cu. ft. of water, and that the solution displaced by it will weigh as much as 1.2 cu. ft. of water. The weight less the loss by buoyancy will be the weight of (2.4 cu. ft. − 1.2 cu. ft. =) 1.2 cu. ft. of water, or 75 lbs.

A comparison of the given densities shows that the coal is twice as heavy as the solution. Under such conditions, the weight of the coal in the solution will equal the weight of the solution displaced.

25. With a force equal to the weight of the iron.

500 g. × 7.8 = 3,900 g.

26.

(1)	Weight of both in air	41.2 g.
(2)	" " " water	26.2 g.
(3)	" lost by both in water . . .	15 g.
(4)	" " iron " "	5 g.
(5)	" " cork " "	10 g.
(6)	Density of cork (2.3 ÷ 10 =)	0.23

27. (*a*) See § 156.

$$11.35 = \frac{600}{600 - w'} = 547.13+.$$

The lead weighs 547.13+ gr. in water.

(*b*) (1) Weight of both in air . . . . 900 gr.
(2) " " " water . . . 472.5 gr.
(3) " lost by both in water . . 427.5 gr.
(4) " " lead " . . 52.87 gr.
(5) " " wood " . . 374.63 gr.
(6) Density of wood (300 gr. ÷ 374.63 gr. = 0.8+.

28.

618 gr.	618 gr.
31 gr.	93 gr.
649 gr. ÷	711 gr. = 0.9+.

29. (*a*) 330 g. − 315 g. = 15 g.
330 g. ÷ 15 g. = 22.
(*b*) 330 g. − 303 g. = 27 g.
27 g. ÷ 15 g. = 1.8.
(*c*) It displaces 15 g. or 15 cu. cm. of water.
Its volume is 15 cu. cm.

30. Its volume must be at least that of 1 Kg. of water. The volume of 1 Kg. of water is 1 liter, 1 cu. dm., or 1,000 cu. cm.

31. See § 160. $2.8 = \frac{37}{37 - w'}$.

$$\therefore w' = 23.785+ \text{ g.}$$

Or, we may say that the body being 2.8 times as heavy as water, an equal bulk of water would weigh $\frac{37}{2.8}$ g. = 13.2142+ g. This is what the body would *lose* in water.

$$37 \text{ g.} - 13.2142 + \text{g.} = 23.785+ g.$$

Page 186.

1. 15 lbs. × 20 × 144 = 43,200 lbs.

2. $\pi D^2 = 3.1416 \times 16 = 50.2656$, the number of square inches of surface. 15 lbs. × 50.2656 = 753.984 lbs.

3. The room contains 6,000 cu. ft. or 10,368,000 cu. in. of air. This weighs 0.31 gr. × 10,368,000 = 3,214,080 gr., or 459.154 lbs. Avoirdupois.

4. If the barometer-tube had a sectional area of 1 sq. cm., the atmospheric pressure per sq. cm. would support a mercury column containing 76 cu. cm. Such a column of water would weigh 76 g.; such a column of mercury would weigh 76 g. × 13.6 = 1,033.6 g.

Each side of the cube has a surface of 100 sq. cm. The six sides have a surface of 600 sq. cm. The atmospheric pressure being 1.0336 Kg. per sq. cm., the total pressure is 1.0336 Kg. × 600 = 620.16 Kg.

5. It loses the weight of a cubic foot of air, or about an ounce and a quarter. § 164.

6. (*a*) The trunk has an horizontal section of $(2\frac{1}{2} \times 3\frac{1}{2} =)$ $8\frac{3}{4}$ sq. ft., or 1,260 sq. in. 15 lbs. × 1,260 = 18,900 lbs., the *downward* pressure on the top of the trunk. If the trunk has a *flat* top, its area will be the same as that of the horizontal section (or bottom) of the trunk; if it has an arched top, the total pressure on the upper surface will be more than here given, the *excess* being lateral pressure, which would not at all interfere with opening the trunk, even if the air was exhausted from it. The *downward* pressure would not be affected by the shape of the top.

(*b*) The *upward* pressure on the under surface of the trunk top is equal to the downward pressure on the upper surface.

7. (*a*) None.

(*b*) See solution to Exercise 4.

8. (*a*) The capacity of the room is 320 cu. m., or 320,000 l. See § 164. 1.3 g. × 320,000 = 416,000 g., or 416 Kg.

(*b*) and (*c*) The surface is 80 sq. m., or 800,000 sq. cm. The atmospheric pressure being about 1 Kg. to the sq. cm., the pressure on each of these surfaces is about 800,000 Kg.

(*d*) The surface is 32 sq. m., or 320,000 sq. cm. The pressure on each of these surfaces is 320,000 Kg.

(*e*) The surface is 40 sq. m., or 400,000 sq. cm. The pressure on each of these surfaces is 400,000 Kg.

(*f*) The total surface of the room is 3,040,000 sq. cm. The total pressure is 3,040,000 Kg.

(*g*) The outward pressure from within is counterbalanced by the inward pressure from without.

9. A liter of hydrogen weighs 0.0896 g.; 10 liters weigh 0.896 g. The balloon and hydrogen weigh 5.896 g. The 10 l. of displaced air weigh 12.93 g.

$$12.93 \text{ g.} - 5.896 \text{ g.} = 7.034 \text{ g.}$$

Page 190.

1. (*a*) Under a pressure of two atmospheres.

(*b*) Under a pressure of half an atmosphere.

2. Under ordinary circumstances, that quantity of air would weigh $1\frac{1}{4}$ ounces. § 164. To get twice that quantity of air into that space, the pressure upon it must be doubled, i.e., the air must be subjected to a pressure of two atmospheres.

3. (*a*) 500 cu. cm., or half a liter.

(*b*) 1 liter. It makes no difference what gas is used.

4. Half of it.

5. (*a*) 15 lbs. + 5 lbs. = 20 lbs. § 168.

(*b*) The pressure is 30 lbs., or 2 atmospheres. The pressure being doubled, the volume will be halved, i.e., 12 cu. in.

6. (*a*) In the short arm. The rising of the barometer indicates an increase of atmospheric pressure. This increase of pressure will push down the mercury in the open arm, and, consequently, push it up in the closed arm.

Page 196.

1. 30 in. $\times$ 13.6 = 408 in. = 34 ft.

2. 28 ft. $\div$ 0.8 = 35 ft.

3. 76 cm. $\times$ 13.6 = 1,033.6 cm. = 10.336 m., the height to which atmospheric pressure will lift water.

(*a*) and (*b*). It cannot, because atmospheric pressure is not sufficient to force the water up to that height.

(*c*) It can, because the water is lifted by muscular or some similar form of energy not subject to the limitations placed upon atmospheric pressure.

4. At the bottom, because the atmospheric pressure being greater, the resultant of the resultant forces (see § 171) is greater there.

5. 34 ft. $\div$ 1.8 = 18.8 ft.

6. The stone is pushed up by the pressure of the air on the under side of the stone, when that pressure is not counterbalanced by the weight of the stone and the downward pressure of the air on the upper side of the stone. That downward pressure of the air is largely carried by the string that carries the sucker.

7. (*a*) $(\frac{4}{5})^4 = \frac{256}{625}$.

(*b*) There being only $\frac{256}{625}$ as much air in the receiver as there was at the beginning, when its elastic force was the same as that of the external air, its density and, hence, its elastic force are only $\frac{256}{625}$ as great as those of the external air.

8. 29.5 in. $\times$ 13.6 $\div$ 1.35 = 223.11 in.

9. 69 cm. $\times$ 13.6 = 938.4 cm. = 9.384 m.

10. (*a*) 15 lbs. $\times$ 3.1416 $\times$ 2^2 = 188.496 lbs.

(*b*) 1 Kg. $\times$ 3.1416 $\times$ 4^2 = 50.2656 Kg.

11. By placing the pump within 28 ft. of the surface of the water and extending the spout (Fig. 140) to the top of the well. The cylinder of the pump may be a tube extending to the top of the well, the piston rod running down the inside of such long cylinder.

12. The elastic force of the air, when you blow in at *f*, lifts the water to *h*, and fills the tube. The tube then constitutes a siphon, and continues to deliver water into *g*. The rising of the water in *g* reduces the air space, and thus increases the elastic force of the air in *g*, *i* and *a*. This increased pressure exerted by the air on the surface of the water in *a* is transmitted to the water contained in *a*, and forces it out in a jet at *n*.

CHAPTER III.

Page 213.

1. See § 183.

2. $1{,}145 \div 458 = 2.5$, the number of feet.

3. The water is a much better conductor of sound than the air is. See § 179.

4. The sound first heard is propagated through the iron; the other, through the air in the pipe. The iron transmits the sound waves more rapidly than the air.

5. The first refers to the vibrations of the sounding body, or to vibrations caused by them in the air or other medium that may carry them to the ear; the second refers to the sensations produced by said waves through the ear or organ of hearing.

6. See § 180 (*a*).

7. In a vibratory motion, each moving particle has a to-and-fro motion so that it periodically returns to the starting point; translatory motion is progressive so that the moving particle does not return to the starting point. The wind may swing a thistlehead to and fro, giving it a vibratory motion; it may also loosen some of the winged seeds, and carry them to another field with a translatory motion.

8. Because the velocity of the particle changes like that of a common pendulum. See Experiment, § 108.

9. Nowise; they are the same.

10. Sinusoidal.

Page 222.

1. 1,090 ft. $\times$ 18 = 19,620 ft.

2. (1,090 ft. + 2 ft. $\times$ 15) $\times$ 7 = 7,840 ft. See § 184 (*c*).

3. Exercise 2 shows that the velocity of sound at the given temperature is 1,120 ft. per second. 1,120 ft. $\div$ 224 = 5 ft. See § 184 (*b*).

4. $\frac{1,150 - 1,090}{2} = 30$, the number of centigrade degrees above the freezing-point, i.e., 30° C.

5. It acts as a reflecting surface to the ear, and cuts off confusing sound waves from sources other than the speaker.

6. The velocity is 1,120 ft. per second. The sound reached the cliff in 3 seconds. 1,120 ft. $\times$ 3 = 3,360 ft.

7. The velocity must be 1,100 ft. per second.

$\frac{1,100 - 1,090}{2} = 5$, the number of centigrade degrees above the freezing-point, i.e., 5° C.

8. 1,120 ft. $\div$ 4 ft. = 280, the number of vibrations. The given velocity implies a temperature of 15° C. See solutions 2 and 3 above.

9. 332 m. = 33,200 cm. 33,200 cm. $\div$ 830 = 40 cm.

10. 1,128 ft. $\div$ 12 ft. = 94, the number of vibrations per second.

11. It took the sound $\frac{3}{16}$ of a second to reach the cliff. The velocity of sound at the ordinary temperature of the air (15° C) is 1,120 ft. $\frac{3}{16}$ of 1,120 ft. is 210 ft.

12. It is a case of echo. It took the sound $1\frac{1}{4}$ seconds to reach the reflecting surface. 1,090 ft. $\times$ $1\frac{1}{4}$ = 1,362$\frac{1}{2}$ ft.

13. Let x represent the depth of the mine. From the third formula in § 107, we derive $t^2 = \frac{2l}{g}$, and $t = \sqrt{\frac{2l}{g}}$. Hence, $\sqrt{\frac{2x}{32.16}}$ represents the time of falling, $\frac{x}{1{,}120}$ represents the time required for the sound to pass from the bottom to the top of the shaft. Adding the times required for these two motions, we have

$$\sqrt{\frac{2x}{32.16}} + \frac{x}{1{,}120} = 5.$$

The algebraic solution of this equation for the value of x, the required depth of the mine, is somewhat tedious, and may well be waived by the teacher, the mere statement of the problem as above being accepted as an answer.

14. See § 184 (*d*). Compare Exercise 4, page 214.

15. The prison was a whispering gallery. See § 185.

16. The usual temperature of 15° C., and the corresponding velocity of 1,120 ft. per second, are assumed. The strokes follow at intervals of $\frac{1}{5}$ of a second; i.e., the pulse caused by any stroke is $\frac{1}{5}$ of 1,120 ft., or 224 ft. in advance of the pulse caused by the succeeding stroke. When the bells are at the same distance from the hearer, the pulses arrive at the listening ear at the same time, and blend as a single pulse caused by a single stroke. When one bell is 224 ft. further away than the other, the first pulse from the further bell arrives at the same time as the second pulse from the nearer bell, and blends with it as a single pulse caused by a single stroke; succeeding pulses from the two bells similarly blend. But when the distances of the bells differ by 112 ft., the first pulse from the further bell arrives $\frac{1}{10}$ of a second after the first pulse from the nearer bell; $\frac{1}{10}$ of a second later, the second pulse from the nearer bell arrives; $\frac{1}{10}$ of a second later, the second pulse from the further bell arrives, etc., in succession, the pulses arriving at intervals of $\frac{1}{10}$ of a second instead of blending in pairs as before. Pulses that arrive at intervals of $\frac{1}{10}$ of a second give the impression of 10 strokes per second.

Page 242.

1. See § 194.

$144 \times \frac{6}{5} = 172\frac{4}{5}$, the number of vibrations for the minor third.
$144 \times \frac{5}{4} = 180$, the number of vibrations for the major third.
$144 \times \frac{3}{2} = 216$, the number of vibrations for the fifth.
$144 \times 2 = 288$, the number of vibrations for the octave.

2. The octave above is produced by $(264 \times 2 =)$ 528 vibrations. The fifth above the octave is produced by $(528 \times \frac{3}{2} =)$ 792 vibrations.

3. The disk revolves $21\frac{1}{3}$ times per second. The disk has 24 holes in the inner row. The number of puffs emitted per second is $24 \times 21\frac{1}{3} = 512$. The unison indicates that the given tone is due to 512 vibrations per second. This is twice the number of vibrations assigned to middle C, so that the tone is just one octave above middle C; i.e., it is C_4. See § 196.

4. Evidently the vibrations of the two forks are as 9 : 15. Referring to the relative vibration-numbers as given in § 196, we see that 27 corresponds to the given lower tone D. The proportion 9 : 15 : : 27 : 45 shows that the higher tone is B.

5. Multiply 261 successively by the vibration-ratios given in § 196.

6. The whistle has a certain vibration-number corresponding to its pitch. When the locomotive and the listener are standing still, the number of pulses that come to the ear is the same as the vibration-number. When the locomotive is approaching the observer, the number of pulses per second that arrive at the ear exceeds the vibration-number, and the pitch is therefore raised. When the locomotive is receding from the observer, the number of pulses per second that arrive at the ear is less than the vibration-number, and the pitch is therefore lowered. The change of pitch is very noticeable if the whistle is blowing when the locomotive passes by the observer.

7. See the relative vibration-numbers in § 196.

$36 : 40 :: x : 440$; $x = 396$, the number of vibrations.

8. Because the resistance of the board reduces the velocity of the saw, and the rapidity of the successive blows of the saw-teeth.

9. The rapidity with which the pulses would strike his ear would be doubled, and the pitch would be raised an octave.

10. The rapidity with which the pulses would strike his ear would be lessened. The tone would be flatted. We might imagine the velocities to differ so little that the number of pulses that overtook the ear per second would be so small that they would be recognized as separate sounds instead of blending into a continuous tone.

11. The observer would overtake in inverse order the pulses that started ahead of him; the music would be heard again as if played backward.

12. The reed has a certain vibration-number. When the observer is in the prolongation of the axis of rotation, his distance from the reed is constant, and the number of pulses that come to his ear per second is the same as the vibration-number of the reed. As the number of pulses does not vary, the pitch is constant. When the observer stands in the plane of rotation, the reed approaches him, and recedes from him once for every revolution of the axis. When the reed is approaching the ear, the succession of pulses is more rapid, and the pitch rises. When the reed is moving away from the ear, the succession of pulses is less rapid, and the pitch falls.

Page 254.

1. See § 202.

2. Three beats per second.

3. The strings are not in unison, or there would be no beats. The vibration-number of the second string is 398 or 402 per second.

4. $308 - 297 = 11$.

5. See § 205 (*a*). As the prong swings outward, it sends a condensation down the jar. If the air column in the jar is $\frac{1}{4}$ of a wave-length, the condensation will travel to the lower end of the air column and back in $\frac{2}{4}$, or $\frac{1}{2}$, the period of the fork. At the end of this half-period, the prong is beginning its swing in the opposite direction, so that the reflected pulse will coincide with the direct outward pulse. If the length of the air column is $\frac{3}{4}$ of a wave-length, the reflected pulse will coincide with the direct pulse of the prong on its second return swing; if it is $\frac{5}{4}$ of a wave-length, the reflected pulse will coincide with the direct pulse of the prong on its third return swing; etc.

6. See § 205 (*d*).

7. (*a*) 15 in. $\times$ 4 = 60 in., or 5 ft.

(*b*) Assume that the temperature is such that the velocity of sound in air is 1,120 ft. $1{,}120 \div 5 = 224$, the number of pulses per second. As the vibration-number is 224, the wave-period is $\frac{1}{224}$ of a second.

8. The velocity of the sound is 34,160 cm. See § 184 (*c*). The wave-length is 64.8 cm. $\times$ 4 = 259.2 cm. $34{,}160 \div 259.2 =$ 131.8, the computed vibration-number. Under the conditions assumed in the text, we must allow for experimental error, and take the nearest whole number for the actual vibration-number.

9. Knowing the velocity of sound per second at a given temperature, and the number of vibrations per second that the sounding body undergoes, it is evident that the wave-length may be determined by dividing the velocity by the vibration-number; i.e.,

$$w = \frac{v}{n}.$$

From § 205 (*b*), we also know that the wave-length equals four times the length of the resonant air column increased by ¼ of its diameter; i.e.,

$$w = 4\left(l + \frac{d}{4}\right).$$

Equating these values of w, we have

$$4\left(l + \frac{d}{4}\right) = \frac{v}{n}; \text{ whence } v = 4\left(l + \frac{d}{4}\right)n.$$

10. $w = 4\left(50 + \frac{4}{4}\right) = 204$, the wave-length in centimeters.

11. There are two beats per second. $512 - 2 = 510$, the vibration-number of the loaded fork.

Page 266.

1. Ignore the number of vibrations, and see § 210.

2. (*a*) The vibration-number is 50. See § 210 (1).

(*b*) The vibration-number is 200.

3. See relative vibration-numbers, § 196, and § 210 (1).

$27 : 24 :: 18 : x$; $x = 16$, the length in inches.

4. $\frac{100}{2 \times \sqrt{4}} = 25$, the vibration-number.

5. See §§ 196 and 210 (2).

$3 : 4 :: \sqrt{4} : \sqrt{x}$; $x = 7\frac{1}{9}$ lbs.

6. 3 ft. : 5 ft. :: 256 : x; $x = 426\frac{2}{3}$, the vibration-number.

7. $2 : 1 :: \sqrt{16} : \sqrt{x}$; $x = 4$, the number of pounds.

8. (*a*) The wave-length = 1,120 ft. ÷ 32 = 35 ft. The length of the air-column is half the wave-length. See § 212 (*a*). 35 ft. ÷ 2 = 17½ ft., the length of the tube.

(*b*) The wave-length = 1,120 ft. ÷ 4,480 = ¼ ft., or 3 in. The length of the air-column is half the wave-length, or 1½ in.

9. (*a*) The length of the air-column in an open pipe is half the wave-length. 4 ft. $\div$ 2 = 2 ft., the length of the open pipe.

(*b*) The length of the air-column in a stopped pipe is a fourth of the wave-length. § 212 (*a*). 4 ft. $\div$ 4 = 1 ft., the length of the stopped pipe.

10. The catgut string will vibrate three times as rapidly.

$$\sqrt{9} = 3. \quad \S\ 210\ (4).$$

11. There would be marked differences in pitch. The velocity of sound is greater for hydrogen than it is for air, and greater for air than it is for carbon dioxide. See § 184. For a given frequency of vibration, the greater the velocity, the greater the wave-length. The length of the air-column has a definite relation to the wave-length. See § 212 (*a*). Consequently, a change of gas that increases the wave-length must be accompanied by an increase in the length of the organ-pipe, or by a rise of pitch. As the velocity of sound in hydrogen is almost four times as great as it is in air, the tone emitted by a pipe filled with hydrogen is nearly two octaves above that produced by the same pipe filled with air. For a similar reason, the tone yielded by the pipe when filled with carbon dioxide is of lower pitch.

12. Drive the handle of a pocket tuning-fork into a hole bored in a block of wood, and incline the fork as shown in the accompanying figure. Suspend a varnished cork-ball, the size of a pea, by a silk fiber or very fine thread, sound the fork and at once bring the ball against the face of a prong. Hunt for a position in which the ball will remain at rest touching the face of the prong.

One or more such places may be found just above the crotch of the fork. These places of rest are nodes. Then slowly raise the ball, keeping it in contact with the fork; it will begin to tremble. Raise it higher until, near the end of the fork, it is driven violently away.

CHAPTER IV.

Page 274.

1. $36 \div 1.8 = 20$, the difference in centigrade degrees.

2. $35 \times 1.8 = 63$, the difference in Fahrenheit degrees.

3. (*a*) $68° - 32° = 36°$; $36° \div 1.8 = 20$, the centigrade reading.
(*b*) 68° F.

4. 10° C.

5. The mercury would stand higher, and the readings would indicate temperatures higher than the real temperatures.

6. The error of the readings would increase as the temperature became higher.

7. $15° \times 0.8 = 12°$, the Reaumur reading.

8. It would indicate that the paint increased the thermal emission of the tin.

9. The expansion of the flask increases its capacity before the contained liquid expands.

10. If the bulb remained as large, a diminution of the bore would increase the sensitiveness of the instrument.

11. Perhaps a little vapor of mercury; otherwise it is a high vacuum.

12. The flow from the warmer to the cooler body will vary with the difference of the temperatures.

13. There are none. See § 222.

14. See § 222 (*a*).

Page 282.

1. Because the cloth is a poor conductor of heat.

2. Because the oil-cloth is a better conductor of heat than the carpet is.

3. Non-conductor in each case.

4. The confined air is a good non-conductor.

5. Because of the convection currents thus set up. See §§ 227 and 228.

6. See answer to Exercise 4. The animal heat is better retained thereby.

7. Copper is a better conductor than tin.

Page 286.

1. See § 230 (*a*). The contraction of the cooling iron forces the parts of the wheel closely together; the cooled iron holds them there.

2. Such walls generally bulge outward. See § 230 (*a*).

3. To provide for the inevitable expansion and contraction caused by variations of temperature.

4. Because the contraction of the alcohol in cold weather exceeds the diminution in the capacity of the measure.

5. See § 228.

6. The surface temperature is 0°. The water at the bottom of the pond at the same time may be 4°. See § 232 (*a*).

7. 273°. See § 232.

12. $273 : 273 + 300 :: 900 : x$.

13. $273 : 273 + 100 :: 1{,}000 : 1{,}366.3$; $1{,}366.3 - 1{,}000 = 366.3$.

14. $273 + 50 : 273 + 15 :: 15{,}000 : x$.

15. $185° \text{ F.} = 85° \text{ C.}$

$273 + 85 : 273 + 10 :: 98 : x.$

16. $\left.\begin{matrix} 273 + 10 : 273 + 18.7 \\ 590 : 530 \end{matrix}\right\} :: 155 : x.$

$x = 143.5 +$, the number of cu. cm.

17. $273 : 273 + 60 :: 231 : x.$

18. The water contracted, and the level in the tube fell as the temperature fell to 4°. As the water cooled from 4° to 0°, it expanded, and the level rose. See § 232 (*a*).

Page 295.

1. By confining the steam, thus increasing the pressure on the liquid and raising its temperature. See § 240 (*a*). A vessel designed for this purpose is called a digester.

2. See answer to Exercise 1.

3. By pumping out the steam as fast as it is formed, and thus reducing the pressure on the liquid. See § 240 (3). A vessel designed for this purpose is called a vacuum-pan.

4. No.

5. It expands or it would not float. It resembles ice in this respect. As it expands in the mold, it fills every part of the mold, in consequence of which the face of the casting is sharply defined. Lead and iron when similarly cast lack the clear-cut outlines of ordinary type.

6. They are due to differences in atmospheric pressure. See § 240 (3).

7. See § 235 (3). The melting-point is lowered by the pressure, and the ice melts at the surfaces of contact. The water thus produced has the temperature of the ice from which it came. When the pressure is removed, the melting-point becomes higher than the temperature of the water, which, consequently, freezes.

8. The air is said to be too dry; i.e., the quantity of watery vapor should be increased as the temperature is increased. This may be done by keeping an open vessel of water on the stove or in the hot-air chamber of the furnace.

9. At 115°. See § 235 (1).

10. See § 241.

11. See § 240 (*c*).

12. See § 240 (*b*).

13. Some of the water passes through the vessel and evaporates. The heat that produces this change of condition disappears in the process. See § 236. This withdrawal of heat lowers the temperature of the vessel and its contents. This exercise and the one following may well be reviewed after the study of latent heat.

14. When the water freezes it gives out heat. See § 234. When the ice melts the heat that does the work disappears in the process. See § 233.

15. They are cases of sublimation. See § 240 (*d*).

16. An increase of the humidity lowers the dew-point.

Page 303.

1. Find the number of calories that may be furnished by the several quantities of water in cooling to any given temperature, as 0° C.

1	Kg.	at	40°	gives	40	large calories.
2	"	"	30°	"	60	" "
3	"	"	20°	"	60	" "
4	"	"	10°	"	40	" "
10	"			"	200	" "

These 200 heat-units would warm the 10 Kg. of water 20° above the assumed temperature, or to 20°. It makes no differ-

ence what temperature is chosen for the reduction. If, e.g., we try a temperature of 10° we shall have $(30 + 40 + 30 + 0 =)$ 100 heat-units. This quantity of heat will warm 10 Kg. of water 10° above the chosen temperature, or to 20°. If a still higher temperature is chosen for the reduction, care being given to the algebraic signs, the result will still be the same.

2. Instead of the water-gram-degree unit, or calory, we may use the water-pound-degree unit. Let x represent the specific heat of the mercury. The mercury was cooled from 20° to 0.634°, or 19.366°. $1 \times 19.366 \times x =$ the number of heat-units given up by the lead; $1 \times 0.634 \times 1 =$ the number of heat-units received by the water. $19.366\,x = 0.634$; $x = 0.0327$.

3. Let x represent the number of pounds of water required. The water will be cooled to 0° and will part with $85\,x$ heat-units. The melting of the ice will require 80×15 heat-units. $85\,x = 1{,}200$; $x = 14.117+$.

4. The specific heat of ice being 0.5, it will require 50 heat-units to warm the ice to the melting-point. It will require 800 more to melt it. $95\,x = 850$; $x = 8.94$.

5. The specific heat of steam is 0.48. Each pound of steam, in cooling to 100°, would yield 12 heat-units. These, with the heat from condensation and cooling to 25°, would yield $(12 + 537 + 75 =) 624$ heat-units. The required quantity of steam must furnish $624\,x$ heat-units. To warm the ice to 0° will require 20 heat-units. This, with the heat required for fusion and warming the melted ice to 25° C., amounts to 545 heat-units.

$$624\,x = 545;\ x = 0.87+.$$

6. The gram of steam will yield $(537 + 100 =) 637$ calories. Each gram of ice will require 80 calories. $637 \div 80 = 7.96$, the number of grams of ice.

7. Assume any quantity of ice, as 1 gram. Let x represent the initial temperature of the water. The melting of the ice requires 80 calories, which must be supplied by 1 gram of water in cooling to 0°. For 1 gram of water to part with 80 calories, it must cool 80°. Consequently, the initial temperature must be 80° higher than its final temperature.

8. Let x represent the temperature. Then, $80 + x = 10\,(20 - x)$; $x = 10.9+$.

9. The water can furnish 60 heat-units for melting ice. This will melt $\frac{3}{4}$ of a pound of ice. The result will be $\frac{1}{4}$ of a pound of ice and $5\frac{3}{4}$ lbs. of ice-cold water.

10. The steam can furnish 6,370 heat-units in cooling to 0°. This heat must warm 1,010 g. of water, and can raise it to $(6{,}370 \div 1{,}010 =)\ 6.3°+$. Or, we may let x represent the temperature. Then $10\,(637 - x) = 1{,}000\,x$; $x = 6.3°+$.

11. Let x represent the resulting temperature. $537 + 100 - x = 10\,x$. $537 + (100 - x) = 10\,x$; $x = 58$ nearly.

12.

	Iron.	*Water.*
Specific heat,	0.1138	1.
Weights,	200.	1,000.
Changes of temperature,	$300 - x$	x

$$6{,}828 - 22.76\,x = 1{,}000\,x;\quad x = 6.67+.$$

13. Let x represent the specific heat. The temperature of the 80 grams falls 80°.

$$6{,}400\,x = 200 \times (20 - 10);\quad x = 0.31+.$$

14. The ice requires 8,000 heat-units. Each pound of steam will supply 637 heat-units. $8{,}000 \div 637 = 12.55$.

15. It will require $(80 \times 5 =)\ 400$ water-ounce-degree heat-units to melt the snow. The water can furnish 460 such units. Hence the snow will all be melted, and the water warmed. Let x represent the resultant temperature of the water.

$$5\,(80 + x) = 23\,(20 - x);\quad x = 2.14°+.$$

16. The steam must supply 5,000 water-pound-degree heat-units. Each pound of steam can supply $(537 + 90 =) 627$ such units. $5{,}000 \div 627 = 7.97$.

17. Let x represent the number of grams of mercury. From Exercise 2, it appears that the specific heat of mercury is 0.0327.

$$150 \times 299 \times 0.0314 = 0.0327\,x; \quad x = 43{,}066.97.$$

18. Let x represent the resultant temperature.

$$4\,[537 + (100 - x)] = 200\,(x - 10); \quad x = 22.29°.$$

19. That the specific heat of sulphur is 0.2.

20. The specific heat of a body represents the number of calories required to raise the temperature of 1 gram of that substance 1 degree, and will be the same whatever the mass of the body. The thermal capacity of the body introduces mass as a factor. See § 248.

21. Much less, on account of the high specific and latent heat of water. As will appear further on (see the answer to Exercise 12, on page 389), the watery vapor in the earth's atmosphere acts as a trap for solar heat, or as a huge terrestrial blanket.

22. Work is done upon the gas; it is liquefied by the pressure put upon it. The latent heat, not being employed in maintaining the aëriform condition, appears as sensible heat. When the liquefied gas expands and resumes its aëriform condition, it does work and the reverse thermal effects follow.

23. See § 234.

24. Work is done upon the air, as in Exercise 22, and the compressed air is heated thereby. The pump is heated by the heated air. Exercises 22 and 24 will be better understood after the section on the relation between heat and work has been studied.

Page 311.

1. Because of this great latent heat of water, the processes of melting ice and freezing water are necessarily slow. Otherwise the waters of our northern lakes might freeze to the bottom in a single night, while "the hut of the Esquimaux would vanish like a house in a pantomime," or all the snows of winter be melted in a single day, with inundation and destruction.

3. The work done by the engine is $(8{,}540 \times 50 =) 427{,}000$ kilogrammeters, or 427,000,000 grammeters. As the mechanical equivalent of a calory is 427 grammeters, the work performed is equivalent to 1,000,000 calories, or to 1,000 large calories.

4. See § 252. $34{,}462 \div 1{,}000 = 34.462$. The number of calories developed would raise the temperature of 1,000 g. of water 34.462°.

5. $80 \times 1{,}390 = 111{,}200$, the number of feet (*in vacuo*) that the ice must fall.

6. See § 247 (*b*). $0.1138 \times 100 = 11.38$, the number of water-pound-degree heat-units required. Each such unit is equivalent to about 1,400 foot-pounds, a total of $(1{,}400 \times 11.38 =)$ 15,932 foot-pounds, or nearly enough to raise 8 tons a foot high.

7. $8{,}080 \times 5 \times 1{,}400 \div 2{,}000 = 28{,}280$, the number of feet.

8. 88.42% of 5 lbs. = 4.421 lbs., the quantity of carbon.
5.61% of 5 lbs. = 0.2805 lbs., the quantity of hydrogen.
$8{,}080 \times 4.421 \times 1{,}400 = 50{,}010{,}352.$
$34{,}462 \times 0.2805 \times 1{,}400 = 13{,}533{,}227.4.$
Total number of foot-pounds, 63,543,579.4.
$63{,}543{,}579.4 \div 2{,}000 = 31{,}771.8$, the number of feet.

9. The weight makes no difference, as an increase in the weight would increase the working power and the work to be done at the same rate. It may be called 1 gram (or anything else).

That weight of water heated 100° would require 100 calories. That weight of lead heated 100° would require ($0.0314 \times 100 =$) 3.14 calories, equivalent to 1,340.78 grammeters. See § 85.

$K.E. = \frac{wv^2}{2g}$; $1{,}340.78 = \frac{v^2}{19.6}$; $v = 162.108$, the velocity in meters.

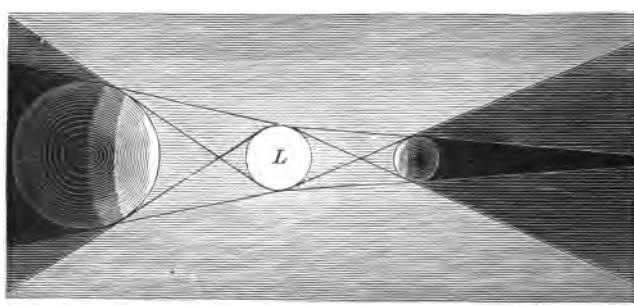
10. $6(442 - 374) \times 0.056 = 22.848$, the number of water-pound-Fahrenheit degree heat-units required for heating the tin to its melting-point. $25.6 \times 6 = 153.6$, the number of such units required for melting the tin. The total of such units required is 176.448. The mechanical equivalent of each such unit is 778 foot-pounds (§ 251). 778 foot-pounds $\times$ 176.448 = 137,276.544 foot-pounds.

CHAPTER V.

Page 323.

1. (*a*) The shadow is the frustum of a cone, the curved surface of which is bounded by rays that are tangent to the ball. There is no penumbra, and the cross-section increases as the distance from the ball increases. See Fig. 233.

(*b*) The rays that come from each luminous point and that are tangent to the ball form a limiting surface as above described. The umbra is the frustum of a cone, and is surrounded by a penumbra. See Fig. 234, and the left-hand end of the accompanying figure. The cross-section increases with the distance from the ball.



(*c*) The shadow, including umbra and penumbra, is the frustum of a cone as before. The umbra is a cylinder. Have each pupil draw a figure resembling Fig. 234, replacing the lamp by a ball at L, with diameter equal to that of the ball, AB.

(*d*) The shadow, including umbra and penumbra, is the frustum of a cone as before. The umbra is a cone with its base at the intercepting ball. Have each pupil draw a figure as for (*c*), but making the ball at L larger than the ball at AB, as in the right-hand end of the figure on this page.

2. Water waves; because the motion of the particles involved in the wave are transverse as it is in the case of light, while in sound they are longitudinal.

3. See § 254.

4. See § 265.

6. The diameter of the shadow will be 5 times that of the coin. The area of the shadow will be 25 times that of the coin.

7. The wall is 100 times as far from the eye as the screen is. The screen has an area of 9 sq. in. 9 sq. in. $\times 100^2 =$ 90,000 sq. in.

8. See Experiment 211. The distance of the lamp is three times that of the candle. The lamp is $(3^2 =) 9$ times as powerful; i.e., it has 9 c. p.

9. See Experiment 212. One is four times as powerful as the other.

10. The spot is translucent, and transmits more light to the eye than the rest of the paper.

11. As the translucent spot transmits more light than the rest of the paper, it reflects less; the same light cannot be transmitted and reflected.

12. There is no penumbra.

13. See answer to Exercise 1 (*d*).

14. The umbra has a finite length when the luminous body is larger than the intercepting body; under other circumstances, its length is infinite.

15. (*a*) It will be in the umbra near the apex.

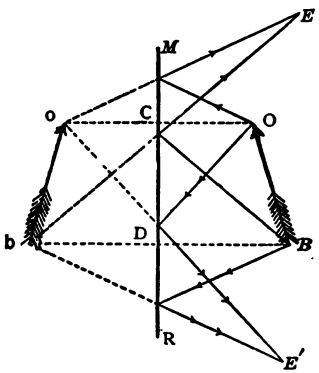
(*b*) It will be in the penumbra at one side of the apex of the umbra.

16. Small density and high elasticity. The formula given in § 184 applies to the medium of any vibratory motion.

Page 340.

1. 45°.

2. In the accompanying figure, the rays that form the image for the right eye are marked with single arrowheads; those that form the image for the left eye, with double arrowheads.



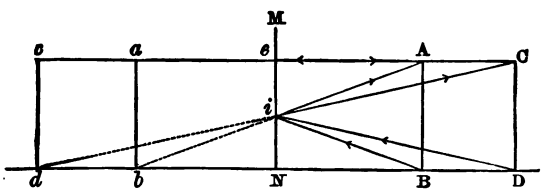
3. (*a*) See § 280 (5);
(*b*) See § 280 (6);
(*c*) See § 280 (4);
(*d*) See § 280 (2);
(*e*) See § 280 (3).

6. See the lines with double arrowheads in Fig. 256.

7. See § 275.

8. (*a*) See § 283; (*b*) see § 281.

9. He cannot in either case. Whatever his position, the image will appear as far back of the mirror as the man is in front of it, and the part of the mirror used to give a complete image will be half the length of the man. If the mirror is not half the length of the man, it cannot give a full-length image of him, no matter what his distance from it. Let *MN* repre-

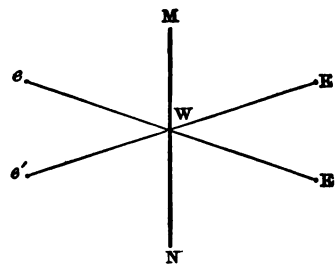


sent the mirror, *AB* the man, and *ab* his image. The triangles, *Aei* and *Aab*, are similar and $Ae = \frac{1}{2} Aa$; therefore, $ei = \frac{1}{2} AB$. But *ei* is the part of the mirror used in forming the image. Suppose now the man to move either way, as to *CD*. The image appears at *cd*, and $Ce = \frac{1}{2} Cc$. Hence, $ei = \frac{1}{2} cd$ or $\frac{1}{2} CD$.

Wherever he stands, he cannot see his complete image unless the length of the mirror is half his own length; if it is too short in one case, it will be too short in every such case.

10. 60°.

11. Let MN represent the mirror, EE' the two eyes, and ee' the two images of the eyes. Of course, EE' are in the same horizontal line. When E' is closed, E sees the image of E' at e'. Placing the wafer at W, hides e'. When E is closed, E' would see the image of E at e, were it not for the wafer which hides it. See § 276. By drawing the parallelogram, $EE'e'e$, and remembering that MN is midway between EE' and ee', it may be proved, geometrically, that Ee' and $E'e$ will intersect at W as represented in the figure.



12. Construct the figure, and measure the lengths of object and image. Modify Fig. 256, by placing OB as far at the left of the mirror as C is at the right.

13. $f' = \frac{fr}{2f - r}$. Either conjugate focal distance equals the product of the other conjugate focal distance into twice the principal focal distance, divided by the difference between twice the other focal distance and twice the principal focal distance. Notice that r is twice the principal focal distance.

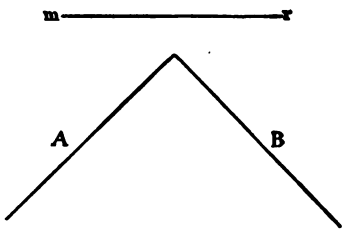
14. Since the sum of the reciprocals of f and f' is constant, i.e., $\frac{2}{r}$, it necessarily follows that if one increases the other must decrease. When either conjugate focal distance, as f, becomes infinite, its reciprocal becomes zero. Substituting this value in the formula given in § 280 (c), we have

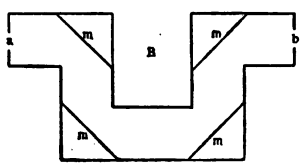
$$0 + \frac{1}{f'} = \frac{2}{r}; \quad \frac{1}{f'} = \frac{2}{r}; \quad f' = \frac{r}{2}.$$

15. Such a negative value indicates that the focus is back of the mirror; i.e., that it is virtual instead of real.

16. Locate the three points on paper, and letter them. Draw the lines, *AC* and *BC*. Bisect the angle, *ACB*, by the line, *CD*. Through *C*, draw the line, *mr*, perpendicular to *CD*. This line, *mr*, indicates the position of the mirror. The angle, *ACD*, is the angle of incidence. The angle, *BCD*, is the angle of reflection.

17. If the middle of a plane mirror is placed opposite the corner of a building so that the line of the face of the mirror makes an angle of 45° with the prolongation of the sides of the house, as shown in the accompanying figure, a person at *A* might see an image of a person at *B*. A rectangular box may





be made with openings, as shown at *a* and *b*, in the second figure accompanying. Mirrors may be placed so as to make angles of 45° with the sides of the box, as shown at *mmmm*. An object at *a* may be seen by an eye at *b*, the light being reflected by each mirror. The experiment is made more interesting by placing an opaque object, as a brick, at *B*.

18. It must be vertical.

19. Two mirrors at an angle of 60° with each other.

20. If the rays that are reflected by the outer parts of the mirror are removed, the foci of the others will not vary much from *F*.

21. Try it with a bit of looking-glass and a pin. The image will be horizontal.

22. See § 270. The small particles of water suspended in the air reflect light, but only those that are in certain positions relative to the observer can reflect light to his eye. The circle sometimes seen around the moon is similarly produced. Compare §§ 303 and 315 (*a*).

Page 363.

1. In the accompanying figure, *ab* represents the plane of the horizon for an observer at *E*. A ray of light coming to the eye from the sun at *S* will be gradually bent, describing a curve. The direction in which the sun is seen is that of a tangent to this curve at the eye. This makes the sun appear higher than its true position, as at *S'*. Similar lines drawn toward the right from *E* will represent the morning phenomenon.

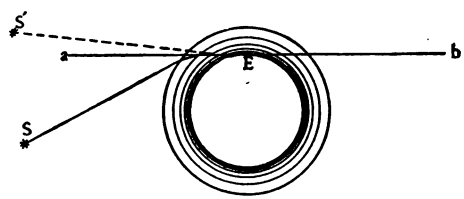


FIG. A.

2. See § 290 (*a*). The pupil is supposed to know that a burning-glass is a single convex lens. If he does not know it, the teacher should send him to the dictionary or other source of information.

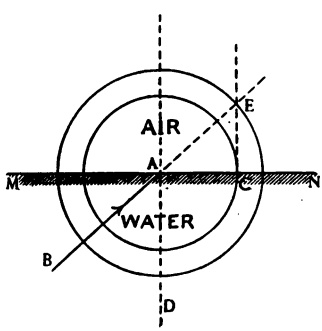


FIG. B.

4. (*a*) See § 285, and Experiments 233 and 234.

(*b*) See Fig. 271, especially the limiting lines, *Aa* and *Ee*.

5. See §§ 285 (*b*) and 284 (*c*). The ratio of the two radii is 4 : 3. The angle *BAD* is the angle sought.

6. See § 288 (*b*).

7. Draw CC' through the centers of curvature. Draw AB, the incident ray, and BC. The problem is to construct BD and DA', the paths of the ray through the lens and beyond. With B as a center and with radii of 2 and 3, draw the arcs, n and m. From the intersection of n and AB, draw a line parallel with BC, and cutting the arc, m. From the point where this line cuts m, draw a line through B to D. The line, BD, is the path of the ray through the lens. Draw DC'. From D as a center and with radii of 2 and 3, draw the arcs, y and x. Produce BD, and from its intersection with x, draw a line parallel to DC' and cutting the arc, y. Through the point where this line cuts y, draw DA', which line marks the path of the ray beyond the lens.

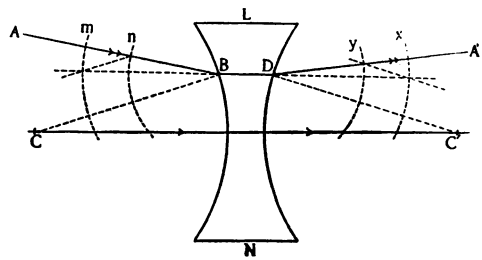


FIG. C.

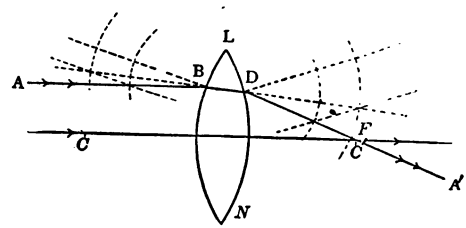


FIG. D.

8. See Fig. D. Draw AB parallel to CC', and complete the construction as in Exercise 7.

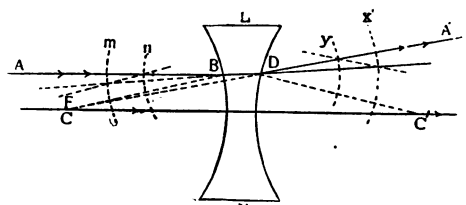


FIG. E.

9. See Fig. E. Draw AB parallel to the principal axis as before, and complete the construction. The focus at F is virtual. See § 292, *b* (1).

10. The optical center.

11. See § 291 (*b*).

(*d*) The method of construction is fully illustrated by Fig. F. The effect of moving the object toward the principal

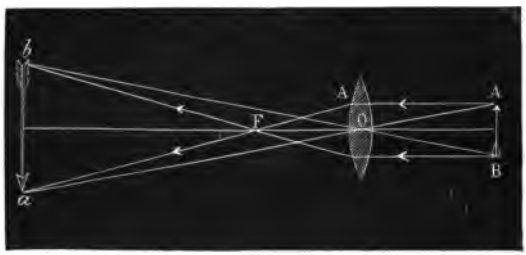


FIG. F.

focus and the lens, the image receding as the object approaches, is illustrated by Fig. G.

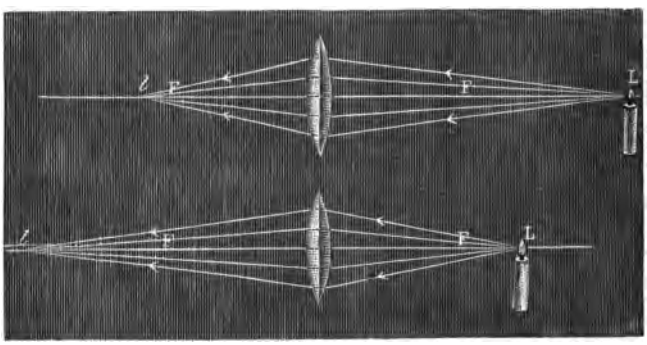


FIG. G.

(*e*) The emergent rays are parallel, as is illustrated by Fig. H, and form no image.

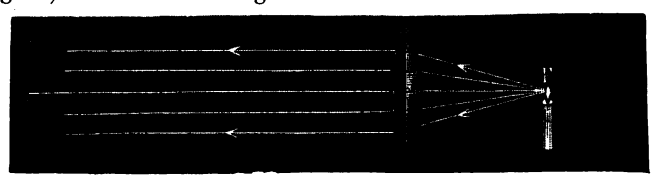


FIG. H.

(*f*) Magnified, erect and virtual, as illustrated by Fig. I.

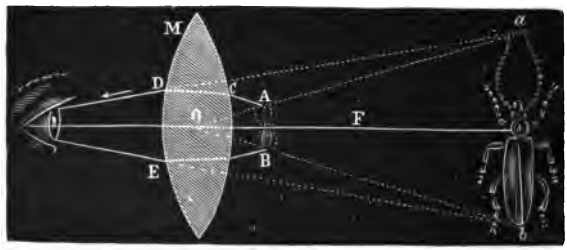


FIG. I.

12. (*a*) See § 291 (*b*) (3).

(*b*) See § 291 (*b*) (2). Construct the image on the scale of 1 : 1.

13. The focus is virtual. See Fig. J.

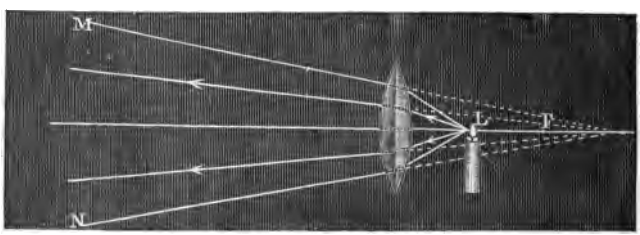


FIG. J.

14. Secondary foci; they will be equal; construct the image.

15. Inasmuch as the incident rays are diverging and the focus is real instead of virtual, it is apparent that the lens is convex. See § 292 (*b*) (2). The focal distance is 8 inches.

16. (*a*) Five feet from the flame. See § 291 (*b*) (4). Notice the principle of reversibility that prevails in optics.

(*b*) One will be five times as long as the other. Their lengths are proportional to their distances from the lens.

17. Make the experiment, and see Experiment 230.

18. Part of the incident light is reflected at each surface of each particle. What escapes reflection by one particle is reflected by some other particle. All of the light being thus reflected, none is transmitted, and the glass is, therefore, opaque.

Page 389.

1. Because of the uniform angular distance from the axis of the bow, as explained in § 303 (*c*).

2. See § 299 (*a*). 0.007×0.3937.

3. $186{,}000 \times 5{,}280 \times 12 \div 0.00002$.

4. See §§ 294 and 298 (*a*).

5. Dispersion. See § 368.

6. See § 303 (*a*).

7. See §§ 307 and 308 (3).

8. See § 308.

9. Because the air is diathermanous; i.e., it transmits the radiant energy instead of absorbing it.

10. The glass is diathermanous to the short-wave radiations that constitute the sunshine, and athermanous to the long-wave radiations emitted by the objects within the greenhouse after they have been heated by absorbing the solar energy.

11. Because the watery vapor in the atmosphere transmits to the earth the short-wave radiations of the sun and absorbs the long-wave radiations from the earth. See § 309 (*b*).

12. Because of the diathermancy of dry air and the athermancy of moist air, as stated in § 309 (*b*). When the night is clear, the earth's heat is rapidly radiated into space; clouds act as a blanket and absorb the energy radiated from the earth, and return some of it to the earth. § 310.

13. The glass is diathermanous to short-wave radiations. As the energy is freely transmitted, it cannot also be absorbed. Only the absorbed energy heats the medium.

14. That water transmits short-wave radiations and absorbs long-wave radiations.

15. "Radiant heat" is used as a synonym for radiant energy, and "obscure heat" as a synonym for radiations of greater wave-length than any that constitute a part of the visible spectrum, neither of which is heat at all. See § 225.

16. If the radiations from the lamp are passed through a solution of iodine in carbon disulphide, the short-wave energy will be absorbed and the long-wave energy transmitted. See §§ 309 (*b*) and 313. If the radiations are passed through a solution of alum in water, the long-wave energy will be absorbed and the short-wave energy transmitted.

17. Because its polished surface is a poor radiator (§ 311).

18. The convection currents of heated air have varying densities and refractive powers.

19. See answer to Exercises 11 and 12.

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10. A Plücker-tube is a Geissler tube (Fig. 409) with a capillary part, by means of which the luminous intensity of feeble electric discharges may be raised sufficiently to allow of spectroscopic investigation.

CHAPTER VI.

Page 430.

1. See Experiment 298.

2. See § 329 (*a*).

3. See § 329.

4. The opposite character of the electrifications is indicated by the attraction between the rubbed body and the body with which it is rubbed; the equality of the two electrifications is indicated by the fact that when the two bodies are brought together, the two electrifications mutually neutralize each other.

5. To avoid the condensation of moisture from the air. See § 326 (*a*).

6. Since the charges are of opposite signs, the force will be attractive and not repellent. See § 332.

$$f = \frac{Q \times q}{d^2} = \frac{24 \times 8}{4^2} = 12.$$

Our units are all C.G.S. units. By recognizing the algebraic signs, we have

$$\frac{+24 \times (-8)}{4^2} = -12,$$

which also indicates an attractive force.

7. The -8 units will neutralize an equal number of the $+$ units, leaving $+16$ units to be equally divided between the two balls (which are assumed to be of equal capacity).

$$f = \frac{Q \times q}{d^2} = \frac{8 \times 8}{4^2} = 4.$$

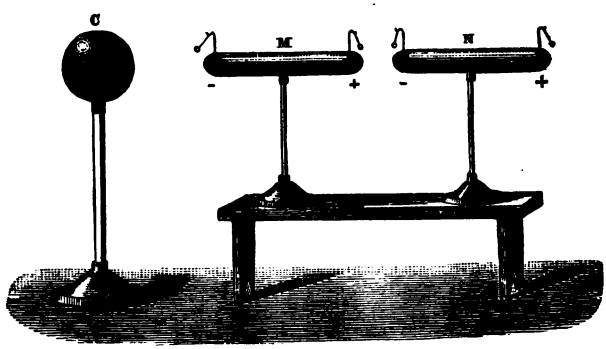
As the algebraic sign of the answer is $+$ (the charges being alike), the force is one of repulsion.

8. $$f = \frac{Q \times q}{d^2}. \quad \therefore 32 = \frac{28 \times 56}{d^2}.$$

$$\text{Whence, } d^2 = \frac{28 \times 56}{32} = 49.$$

$$d = 7.$$

9. (*a*) See Fig. 321. Place the dozen globes in actual contact. They will be polarized as a single body, and may all be charged as described in Experiment 306. (*b*) Charge one negatively by induction; with it, charge another positively by induction. For example, give a positive charge to *C* and bring it near *M*, thus polarizing the latter as shown in the figure.



Touch *M* with the finger, thus leaving it negatively charged. Remove *C*, and bring *N* into position, as shown in the figure. *N* will be polarized by the negative charge of *M*. Touch *N* with the finger and its negative electrification will be repelled to the earth, leaving the conductor positively charged.

10. $\frac{5-3}{2} = 1.$

11. By bringing the charged conductor into simultaneous contact with two conductors of equal capacity.

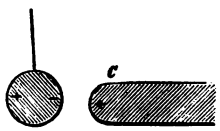
12. See § 333 (*a*).

13. See § 343 (*a*).

14. The shock will be greater when the jar is held in the hand. When the outer coat is insulated, its charge "binds" part of the charge of the inner coat.

15. The divergence will be greater when the jar is held in the hand, for the reason just given.

16. When an electrified glass rod is brought near an electric pendulum, the pith-ball is polarized, as shown in the figure. As the distance between the opposite electrifications is less than the distance between the similar electrifications, the attraction is greater than the repulsion. If the pith-ball is suspended, not by a silk thread but by some good conductor, the attraction will be more marked, for the + of the ball will escape to the earth through the support, and thus the repelling component will be removed.



Page 442.

1. The table gives it as 1.023 ohms.

2. The resistance of 1,000 feet of No. 4 copper wire is 0.254 of an ohm; that of 800 feet of such wire is 0.1932 of an ohm. Multiplying this by the ratio between the resistivities of copper and of German-silver as given in the Appendix (page 592), we have $0.1932 \times 128.29 \div 10.45 = 2.374$, the number of ohms.

3. The resistance of that length of No. 8 copper wire is 0.643 of an ohm $\times 0.750 = 0.48225$ of an ohm. Multiply this by $\frac{63.35}{10.45}$, the ratio between the resistivities of copper and of iron.

4. The resistance of that length of No. 14 copper wire is 2.585 ohms $\times 0.35 = 0.90475$ of an ohm. Multiply this by $\frac{9.84}{10.45}$, the ratio between the resistivities of copper and of silver.

5. 33.135 ohms $\times$ 6.050 = 200.46675 ohms.

6. The table shows that 386.8 ft. of No. 14 copper wire has a resistance of 1 ohm. The given resistance of 1.75 ohms indicates that the fault would be 386.8 ft. $\times$ 1.75 = 676.9 ft. from the end of the line if the wire was of copper. Correct this result by multiplying 676.9 ft. by $\frac{10.45}{63.35}$, the ratio between the resistivities of iron and of copper.

7. The resistance of 3,590 ft. of No. 14 copper wire is 2.585 ohms $\times$ 3.59 = 9.28 ohms. Multiply this by $\frac{63.35}{10.45}$, the ratio between the resistivities of copper and of iron.

8. The length of a No. 14 copper wire that has a resistance of 1.75 ohms is (386.8 $\times$ 1.75 =) 676.9 ft., as given in the answer to Exercise 6.

9. The table shows that the resistance of No. 13 copper wire is very nearly 2 ohms per thousand feet, and that the diameter of such a wire is 0.071961 of an inch, or 1.828 millimeters. For exact determination, apply the principle set forth in § 352 (2):

$$2 \text{ ohms} : 2.048 \text{ ohms} :: (1.828)^2 : x^2.$$

The value of x will represent the diameter in millimeters.

$$2 \text{ ohms} : 2.048 \text{ ohms} :: (71.961)^2 : y^2.$$

The value of y will represent the diameter in mils.

A shorter solution may be had by using the formula for resistivity, given on page 592:

$$2 = \frac{10.45 \times 1{,}000}{d^2}; \quad d = 72+ \text{ mils.}$$

The difference in the two results is partly due to the fact that the resistivity as given in the table was computed for the temperature of 20°, while the resistance was computed for the temperature of 24°.

10. See the formula for resistivity given on page 592.

$$46.2 = \frac{900\,K}{50^2}; \quad K = 128.3.$$

This resistivity indicates German-silver as the metal.

Page 450.

5. See § 81 (*a*).

746 watts = 1 H.P. = 33,000 foot-pounds per minute.

$33{,}000 \div 746 = 44.236.$

6. $9.6 \times 2{,}900 \div 746 = 37.32.$

7. $37.32 \div 0.90 = 41.46.$

8. (*a*) $C = \frac{E}{R}$; $21 = \frac{1.5}{R}$; $R = 0.07143.$

(*b*) $21 \times 1.5 = 31.5.$

(*c*) There will be 1,600 ft. of line wire, the total resistance of which is indicated by the answer to (*a*). This represents a resistance per 1,000 ft. that is less than that of the largest wire mentioned in the table.

9. (*a*) $110 \times 0.5 \div 16 = 3.4375.$

(*b*) $45 \times 10 \div 2{,}000 = 0.225.$

10. There is no difference:

$$900 \times 10 = 1{,}800 \times 5.$$

11. $110 \times 0.5 \times 50 \div 0.90 \div 746 = 4.09.$

12. See § 361 (*b*); $W = R \times C^2 = 23 \times 3.5^2 = 281.75.$

13. $W = \frac{E^2}{R} = \frac{110^2}{37} = 327.02.$

14. (*a*) $\frac{350{,}000}{10{,}000} = 35$, the number of amperes.

(*b*) $35 \times 14 = 490$, the number of volts.

(*c*) $490 \times 35 = 17{,}150$, the number of watts.

(*d*) The 525 H.P. expended on the dynamo is equivalent to $(746 \times 525 =)$ 391,650 watts.

$350{,}000 \div 391{,}650 = 0.893+.$ *Ans.* 89.3+ per cent.

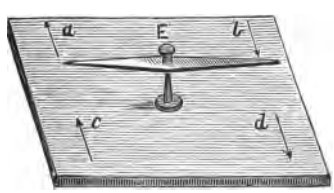
Page 476.

1. The non-active part that is about midway between the poles. See § 367.

2. See § 369 (*d*).

3. By the rapidity of oscillation after the needle has been displaced from its chosen position. The greater the intensity of the field, the greater the rapidity of oscillation. "Whenever a body oscillates under the action of a force, the square of the time of a single oscillation is directly proportional to the moment of the inertia of the body about the axis of oscillation, and inversely proportional to the directive force." As the number of oscillations is the reciprocal of the time or period of an oscillation, it follows that the directive force is directly proportional to the square of the number of oscillations. Compare the formula in § 115.

4. See Laboratory Exercise 2, on page 476. No magnet can be found that tends, as a whole, to drift toward the north or south. The attraction of the north pole of the earth for the marked pole of the needle is counterbalanced by its repulsion for the unmarked pole of the needle. The accompanying figure shows the way in which the needle is placed in a north and south line. The arrow *a* represents the attraction of the north magnetic pole of the earth, and *c* represents the repulsion of the south magnetic pole of the earth acting upon the marked pole of the needle; *b* represents the repulsion of the north pole, and *d* represents the attraction of the south pole of the earth acting upon the unmarked pole of the needle. The combined effect of these four forces is to place the needle in the magnetic meridian. When the needle is thus placed, since the two ends of the needle are practically equidistant from either pole of the



earth, $a + c = b + d$. The two couples thus counterbalance each other.

5. See § 376. $220 \times 3 \div 0.7958 = 829+$.

6. (*a*) $40 \times 5 \div 0.7958 = 251.3$, the number of gilberts.
   (*b*) $251.3 \div 0.00593 = 42{,}378$, the number of webers.
   (*c*) See § 369 (*a*). $43{,}378 \div 3 = 14{,}126$, the number of gausses.
   (*d*) See § 378. $\frac{1}{1684} = 0.000593$, the reluctivity.
   (*e*) $251.3 \div 30 = 8.37$, the number of gausses.

Page 482.

1. (*a*) $C = \dfrac{E}{\frac{r}{n} + R} = 2 \div \left(\dfrac{0.5}{5} + 0.1\right) = 10.$

   (*b*) $C = \dfrac{E}{\frac{r}{n} + R} = 2 \div \left(\dfrac{0.5}{5} + 500\right) = 0.004$ nearly.

2. (*a*) $C = \dfrac{nE}{nr + R} = 10 \div (2.5 + 0.1) = 3.846+$.

   (*b*) $C = \dfrac{nE}{nr + R} = 10 \div (2.5 + 500) = 0.0199+$.

3. $C = 2 \div (1 + 16) = 0.1176$.

4. $C = 16 \div (64 + 16) = 0.2$.

5. $C = 8 \div (16 + 16) = 0.25$.

6. $C = 1 \div 5.001 = 0.19996+$.

7. $C = 1 \div (0.5 + 0.001) = 1.996+$.

8. $C = 10 \div (50 + 0.001) = 0.19999+$.

9. $C = 1 \div (5 + 1{,}000) = 0.00099502$.

10. $C = 1 \div (0.5 + 1{,}000) = 0.0009995$.

11. $C = 10 \div (50 + 1{,}000) = 0.00952$, the number of amperes.

NOTE. — Compare the results in Exercises 6, 7, and 8, where we have a small external resistance. Then compare the results in Exercises 9, 10, and 11, where we have a high external resistance.

12. (*a*) Consult the table in the Appendix. $0.643 \times 0.006 = 0.00386$; $0.8 \times 6 + 0.00386 = 4.80386$.

(*b*) $C = 12 \div 4.80386 = 2.498$.

13. (*a*) $\frac{0.8}{6} + 0.00386 = 0.13719$.

(*b*) $C = 2 \div 0.13719 = 14.6$ nearly.

14. The pupil must judge from the evident analogy, "tandem" referring to grouping in series, and "abreast" referring to grouping in parallel.

15. Test them with a high resistance.

16. Apply the "rule of thumb" given in Experiment 347, and the "cork-screw rule" given on page 468 to determine the direction of the lines of force. Consider the rectangle as a disk magnet (see Experiment 348), and compare its action with that of a magnetic needle. Consider the solenoid as a bar magnet, determine its polarity, and see § 371.

Page 496.

1. See § 388.

2. See § 395.

3. See Experiments 315 and 347.

4. See Experiments 353 and 357.

5. The cast iron has a residual magnetism that is helpful in starting the machine (§ 396*a*), and it is cheaper and more easily worked than wrought iron or pure soft iron is. In most other respects, a pure soft iron core would be vastly preferable.

6. It may depend upon the intensity of the magnetic field, the number of coils on the armature, or the rapidity of revolution of the armature.

7. The field magnets of one are permanent; those of the other are temporary.

8. It diminishes it, by increasing the resistance of the wire.

9. See Fig. 388.

Page 518.

1. (*a*) See § 401. (*b*) and (*c*) See § 400 (*a*).

2. See § 407.

3. See § 400. It weakens the direct current so that it may be unable to force its way through the dielectric, as in the case of the sparks from an induction coil.

4. (*a*) See §§ 378 and 379; oersteds $= \frac{\text{gilberts}}{\text{webers}}$.

(*b*) Draw a right-angled triangle; mark the hypotenuse "impedance," and the other two sides "reactance" and "resistance." See § 401 (*a*).

5. See Fig. 399 and § 403 (*b*).

6. See § 404.

Page 536.

1. See Fig. 425. Represent the current that flows through nx by the letter c; and the current that flows through mp by c'. See § 361; $E = C \times R$.

The fall of potential from A to $C = cn$; represent it by v.

The fall of potential from A to $D = c'm$; represent it by v'.

But when the resistances are adjusted so that there is no flow through G, C and D are at the same potential; i.e., $v = v'$.

Hence, $cn = c'm$, and $\frac{n}{m} = \frac{c'}{c}$.

Similarly, it may be proved that $\frac{x}{p} = \frac{c'}{c}$. Equating these values of $\frac{c'}{c}$, we have $\frac{n}{m} = \frac{x}{p}$, or $m : n :: p : x$. Q.E.D.

2. That they may absorb only a small part of the energy. If either instrument absorbed much, it would change the value of the function that it is to measure. A voltmeter receives the full voltage; it should use a small current; i.e., the resistance should be high. An ammeter receives the full current; its resistance should be low so that the voltage required to force the current through it may not be excessive.

3. (*a*) $C = \frac{E}{R} = \frac{37}{26{,}000} = 0.001423$, the number of amperes.

(*b*) $E = CR = 0.003 \times 26{,}000 = 78$, the number of volts.

4. (*a*) $C = \frac{E}{R} = \frac{110}{25{,}000 + 15{,}000} = 0.00275$, the number of amperes.

(*b*) 40,000 : 25,000 :: 110 : 68.75, the number of volts.

(*c*) 40,000 : 15,000 :: 110 : 41.25, the number of volts.

5. (*a*) $C = \frac{E}{R} = \frac{110}{3{,}500 + 24{,}000} = 0.004$, the number of amperes.

(*b*) $E = CR = 0.004 \times 3{,}500 = 14$, the number of volts.

(*c*) $110 \div 14 = 7.85$.

(*d*) $117 \div 27{,}500 = 0.004254$, the number of amperes.

(*e*) The voltmeter indicates ($E = CR = 0.004254 \times 117 =$) 14.89 volts. $117 \div 14.89 = 7.85$, the same multiplier as before.

6. $(10 - 1) \times 3{,}500 = 31{,}500$, the number of ohms.

7. $110 \times 0.002 = 0.22$, the number of watts.

8. 6.435 : 0.065 :: 6 : x; $x = 0.0606$ of an ohm.

9. Pass a current of known strength through the coil to be measured. Take the readings of a voltmeter at the terminals of the coil. Substitute the known values in the formula, $R = \frac{E}{C}$, and solve for the value of R.

10. See Fig. 387. Connect the terminals of the voltmeter to the main-circuit wire each side of the field magnets.

Page 575.

1. $C = \frac{E}{R} = \frac{838.44}{4.56 \times 16 + 10.55} = 10.04$. The problem ignores the resistance of the line, i.e., assumes that the circuit is short and of inconsiderable resistance.

2. See § 400.

3. See § 414 (*b*).

$$9.925 : x :: \tan 60° : \tan 74°.$$
$$9.925 : x :: 1.73 : 3.49;\ x = 20+.$$

4. $R = \frac{E}{C} = \frac{30}{1} = 30$, the number of ohms.

5. (*a*) $C = \frac{E}{R} = \frac{110}{240} = 0.46$ nearly.

(*b*) $110 \times 0.46 = 50.6$.

(*c*) $50.6 \div 16 = 3.16$.

6. The resistance of the series of lamps will be 250 ohms. That of the wire may, then, be 5 ohms. This is at the rate of 25 ohms per 1,000 ft. No. 24 is the nearest to the size desired, but as the line resistance "must not be more" than 5 ohms, we must use the next larger size, or No. 23, copper wire.

7. The resistance of the lamp circuit will be 2.5 ohms, and that of the 200 ft. of wire 0.05 of an ohm. This is at the rate of 0.25 of an ohm per 1,000 ft. The difference between this desired resistance and that of 200 ft. No. 4 of copper wire is inconsiderable and may be ignored. In practice, a difference like that of Exercise 6 would probably be ignored, and No. 24 wire used.

8. $E = CR = 10 \times 3.8 = 38$.

9. The total resistance of the lamp, including the arc, is 3.8 ohms + 0.62 of an ohm = 4.42 ohms.

$$E = CR = 10 \times 4.42 = 44.2.$$

10. $206 \div (1.6 + 25.4) = 7.63$, the number of amperes.

11. $(2.8 + 1.1 + 9.36) \times 14.8 = 196.248$.

12. It is not running fast enough. With the given resistances, a 25-ampere current will require an E.M.F. of 212.5 volts. The dynamo must be "speeded up" so as to give the additional 12.5 volts.

13. $81.58 \div 29.67 = 2.75$, the total number of ohms.

2.75 ohms $- 1.14$ ohms $= 1.61$ ohms.

14. The total resistance of the circuit is $(157.5 \div 17.5 =)$ 9 ohms. 9 ohms $- 4.58$ ohms $= 4.42$ ohms.

15. $(39.3 \times 3 + 11.2) \times 1.2 = 154.92$.

16. As the lamps were in series, the strength and E.M.F. of the current must have been the same in both lamps.

$$97 \times 2 + 12 = 206, \text{ the number of ohms.}$$

$$E = CR = 1 \times 206 = 206, \text{ the number of volts.}$$

17. The armature wire must carry 4.8 amperes. $4.8 \div 2{,}000 = 0.0024$, the required area in square inches. The nearest size, as given in the fifth column of the table on page 593, is No. 15.

18. The E.M.F. required to overcome the resistance of the lamps is 45 volts $\times$ 60 = 2,700 volts.

The E.M.F. required to overcome the other resistance is $9.6\,(16 + 5) =$ 201.6 volts.

Total voltage required, 2,901.6 volts.

19. $\dfrac{7{,}200{,}000 \times 120 \times 780}{60 \times 100{,}000{,}000} = 112.32.$

20. It would lessen it.

21. The armature that has twice as many bobbins will develop twice as great an E.M.F.

22. As the external resistance is lessened, the current will be increased; if such increase is continued too far, the current will become greater than the No. 8 wire can safely carry, the wire will become red-hot, and its insulation destroyed; i.e., the armature will "burn out." Hence the danger of short-circuiting a dynamo.

23. An increase of the resistance in the field circuit of a shunt dynamo lessens the current that passes around the field

magnets, weakens the magnetic field, and lessens the number of lines of force that pass through the armature. The E.M.F. of such dynamos is often thus controlled.

24. To send a 2-ampere current through a 2-ohm coil requires a pressure of 4 volts; $E = CR$. The dynamo must be run at a speed that will give this voltage. The added resistance must be such that $(9.6 - 2 =) 7.6$ amperes will flow through it at this pressure.

$$R = \frac{E}{C} = \frac{4}{7.6} = 0.526;$$

or $$7.6 : 2 :: 2 : x; \quad x = 0.526.$$

25. (*a*) $110 \div 0.96 = 114.58+$.

(*b*) The fall of potential in the line $= 114.58$ volts $- 110$ volts $= 4.58$ volts.

Current required $= 127 \times 0.5 = 63.5$, the number of amperes.

$$\frac{4.58}{63.5} = 0.0721+, \text{ the number of ohms.}$$

(*c*) $0.0721 \div 1{,}240 = 0.0000581$, the number of ohms.

(*d*) No. 000, i.e., "three naught" wire.

26. $R = \frac{C}{E}$. The resistance of the wire must be $(110 \div 15 =)$ 7.333 ohms. The table shows that the resistance of a foot of No. 14 wire is 0.002585 of an ohm. $7.333 \div 0.002585 = 2{,}837$, the number of feet.

27. (*a*) The two miles of wire have a resistance of

$$(0.051 \times 5.28 \times 2 =) 0.53856 \text{ of an ohm.}$$

(*b*) $175 \div 350 = 0.5$, the number of amperes required for each lamp.

$0.5 \times 100 = 50$, the number of amperes required for the 100 lamps.

$E = CR = 50 \times 0.53856 = 26.928$, the fall of potential or loss of voltage due to the resistance of the wire.

$110 + 26.928 = 136.928$, the voltage required at the dynamo.

28. As either sounder adds little relatively to the high resistance of the line, the current strength is nearly as great with the high resistance sounder in the circuit as it is with the other. On the other hand, the difference in the number of the turns of the wire does materially affect the ampere-turns (§ 376) in favor of the high resistance instrument. Do not let the pupil get the idea that the high resistance of the instrument is the cause of the greater efficiency. The resistance is an incidental and not a desirable factor in the case, but in order to get the great number of turns in a small space, a long fine wire is necessary, and this involves a high resistance.

29. See § 424.

30. The difficulty in the latter case is due to the alternating nature of the telephonic current, and to the greater resistance and impedance of the circuit.

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